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PERFORMANCE OF THE MMPF ALGORITHM FOR DOA ESTIMATION UNDER SENSOR FAILURES

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ABSTRACT

In this paper a recently proposed algorithm for the problem of estimating the direction of arrival angles of narrowband signals emitted from multiple sources is compared to two established ones, namely the conventional beamforming and MUSIC algorithms, through extensive simulation. The emphasis is in assessing the algorithms' performance in adverse situations including availability of a small number of sensors due to sensor failures, and presence of several sources.

Keywords: State estimation, system identification, Kalman filtering, direction of arrival estimation.

1. INTRODUCTION

In this paper we address the problem of estimating the direction of arrival (DOA) angles of narrowband signals emitted from multiple sources, based on measurements obtained by a linear sensor array. This problem has been the focus of substantial research effort during the last two decades due to its importance in several application areas, including sonar and radar signal processing [1], mobile communications [2], acoustics and speech processing [3], structural health monitoring [4], vehicular technology [5], and security monitoring (incident detection and localization).

Several of the proposed solutions employ the Maximum Likelihood (ML) approach in some form, either stochastic or deterministic [6], [7]. A drawback of these approaches is their high computational load. Suboptimal techniques with reduced computational requirements that have become quite popular include the Minimum Variance (MV), MUSIC, Minimum Norm, ESPRIT, and weighted subspace fitting (WSF) algorithms. A key assumption made by all the above methods is that the number of sources that contribute to the received signal is known. In practical situations, however, this prior knowledge is not available. For example, in sonar applications, the number of targets tracked is both unknown and time-varying, as new contacts are occasionally detected. Since the number of sources directly affects the signal model, the DOA estimation problem is in fact a combined estimation and system identification problem. The identification of the number of sources is typically carried our separately by information theoretic methods, such as the Final Prediction Error (FPE) criterion, the Akaike Information Criterion (AIC) [8] and the Minimum Description Length (MDL) Criterion [9]. The techniques that result from the above criteria do not guarantee convergence to the correct model, exhibiting model overfit or underfit. They also require large sets of measurements, which inhibits real-time operation, and cannot address adequately the case of a variable number of sources.

An algorithm that addresses both problems of identification of the number of sources, and estimation of the corresponding directions of arrival has been recently proposed [10]. The algorithm is based on a bank of Extended Kalman Filters (EKFs), each of which is implemented assuming a particular constant number of sources being present. A Multi-Model Partitioning Filter (MMPF) is then used to select the correct model and corresponding EKF.

The DOA estimation problem is further complicated by the following issues:

a) In the presence of a large number of sources, even if this number is both constant and known, it becomes increasingly difficult to provide accurate estimates for all sources.

b) In many cases, the estimation of the DOA angles has to be carried out within a very limited time frame. Thus estimation algorithms have to provide estimates using a small number of observations (snapshots).

c) Most importantly, a system might have to operate with a reduced number of sensors due to failures. Depending on the application, typical configurations involve arrays of between 5 and 10 sensors. In such cases, the loss of a couple of sensors may be critical for system performance.

The aim of this work is to investigate by simulation the impact of the above complications on the MMPF in comparison to two established DOA estimation algorithms, namely the conventional beamforming algorithm [11], and MUSIC [12]. In the following section the problem of DOA estimation is stated and mathematically formulated. The MMPF is briefly presented in Section 3. Section 4 presents our extensive simulation experiments and the performance assessment of the algorithms simulated. Finally, Section 5 summarizes our conclusions.

2. PROBLEM STATEMENT

We assume a linear array comprising m isotropic sensors, which receive the signals emitted from n far-field point sources. Let φ_i be the direction of the i-th source, and $s_i(t)$ the complex envelope of the corresponding received signal. We assume for the moment that the sources are stationary, i.e. the φ_i's are constant. We further assume that the signal characteristics are invariant in time and

$$s_i(t) = e^{j\omega_i t}, \quad i = 1, \ldots, n \tag{1}$$

We define the $m\times1$ vector $a(\varphi_i)$ to be the complex array response to a unit waveform from a source at direction φ_i. Assuming that n signals are simultaneously intercepted, under the narrowband assumption, the array output $z(t)$ is given by the following equation:

$$z(t) = A(\varphi)s(t) + v(t) \tag{2}$$

where

$$z(t) = [z_1(t)\ z_2(t) \ldots z_m(t)]^T$$

$$A(\varphi) = [\alpha(\varphi_1)\ \alpha(\varphi_2) \ldots \alpha(\varphi_n)]^T$$

$$\varphi = [\varphi_1\ \varphi_2 \ldots \varphi_n]^T$$

$$s(t) = [s_1(t)\ s_2(t) \ldots s_n(t)]^T$$

The $m\times1$ vector process $\{v(t)\}$ represents additive noise with variance R and the columns of the $m\times n$ matrix $A(\varphi)$ are the array propagation vectors $a(\varphi_i)$, $i = 1, \ldots, n$. These vectors can be determined by considering the geometry of the problem, as depicted in Fig. 1.

Let d be the spacing between sensors. When the signal wavefront from the i-th source reaches a sensor, the distance to the next sensor down the array is $d\sin\varphi_i$. Thus the time delay between two consecutive sensors is $\tau_i = (d\sin\varphi_i) / c$, where c is the propagation velocity. The corresponding phase difference is $\omega_i\tau_i = (2\pi d\sin\varphi_i)/\lambda_i$, where λ_i is the signal wavelength and ω_i is the angular frequency. The array response $a(\varphi_i)$ for a source at direction φ_i and wavelength λ_i becomes:

$$a(\varphi_i) = \left[1, e^{-j\omega_i\tau_i}, e^{-j\omega_i 2\tau_i}, \ldots, e^{-j\omega_i(m-1)\tau_i}\right]^T \tag{3}$$

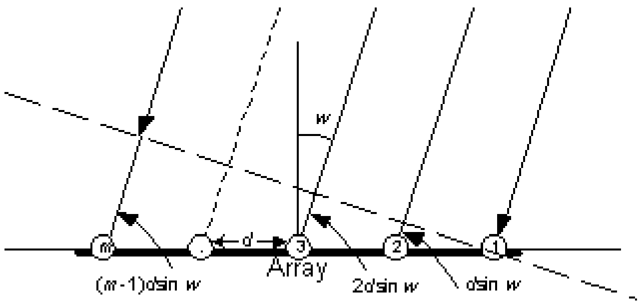

Fig. 1. Arrival delays of wavefront across the array

The output of the l-th sensor, where $l = 1, \ldots, m$, can be written using (2) and (3) as follows:

$$z_l(t) = \sum_{i=1}^{n} e^{j\{2\pi ct/\lambda_i - 2\pi(l-1)d\sin(\varphi_i)/\lambda_i\}} + v_l(t) \tag{4}$$

In order to formalize the problem in a state space form, we choose the following state vector:

$$x = [\lambda_1, \lambda_2, \ldots, \lambda_n, \varphi_1, \varphi_2, \ldots, \varphi_n]^T$$

Under the assumption of stationary sources and observer, the system state x is constant. However, in the general case of moving sources (e.g. in tracking applications), the state equation will be nonlinear and complex:

$$x(k+1) = f[x(k)] + w(k) \tag{5}$$

The array output is sampled at distinct time instants, producing the following sequence of measurements:

$$z(k) = h[x(k)] + v(k) \tag{6}$$

where h is the nonlinear function in (4). Equations (5) and (6) define a nonlinear discrete-time state-space model. The objective is, at each time step k, to obtain an estimate $\hat{x}(k \mid k)$ of the state $x(k)$, given the set of measurements $z(k)$ up to and including time k.

3. A MULTI-MODEL PARTITIONING ALGORITHM FOR DOA ESTIMATION

Assuming that the number of sources n is a known constant, the state space model given by (5)-(6) is completely specified. An Extended Kalman Filter (EKF) can therefore be employed to process the measurements $z(k)$ and obtain recursively the state estimates $\hat{x}(k \mid k)$. The EKF is a suboptimal estimator, since it approximates the nonlinear equation (6) by a linear one. However, its main advantage is that it computes the estimates in real time, without requiring a large batch of data. When the actual number of sources n

differs from the one assumed by the EKF, the algorithm will exhibit large errors, due to the model mismatch.

In practical applications, n is an unknown parameter, although we may be able to set an upper bound $n \le n_{MAX}$. In such cases a viable approach is to employ a bank of EKFs, operating in parallel and independent of each other [10]. Each filter is implemented based on the same set of equations (5)-(6), but assuming a particular value of n. All filters operate concurrently on the same measurements $z(k)$; however, since they assume different system models, each filter produces its own model-conditional estimate $\hat{x}(k \mid k; n)$.

In order to select the correct model and corresponding filter among the n_{MAX} candidate models, we employ a multi-model partitioning algorithm. The general form of the algorithm [13] can produce a minimum-variance state estimate from a (possibly infinite) set of model-conditional estimates. The algorithm is based on the calculation of the a posteriori probability of each model being the correct one. In the problem at hand, the a posteriori probability $p(n \mid k)$ of the parameter n can be recursively calculated as follows:

$$p(n \mid k) = \frac{L(k \mid k; n)}{\sum_{i=1}^{n_{MAX}} L(k \mid k; i)\, p(i \mid k-1)}\, p(n \mid k-1) \tag{7}$$

where $L(k \mid k; n)$ is a likelihood function given by

$$L(k \mid k; n) = \left| P_{\tilde{z}}(k \mid k-1; n) \right|^{-1/2} \times \exp\left\{ -\tfrac{1}{2} \tilde{z}^T(k \mid k-1; n) P_{\tilde{z}}^{-1}(k \mid k-1; n) \tilde{z}(k \mid k-1; n) \right\} \tag{8}$$

The quantities $\tilde{z}(k \mid k-1; n) = z(k) - H(k; n)\hat{x}(k \mid k-1; n)$ and $P_{\tilde{z}}(k \mid k-1; n)$ are the conditional innovation sequences and corresponding covariance matrices produced by the conditional EKFs. Finally, $H(k; n)$ are the observation matrices produced by the filters during the linearization of the nonlinear function h in (6).

At each step k, the algorithm selects as the number of sources the value of n that maximizes the a posteriori probability $p(n \mid k)$. The DOA estimates are then given by the conditional estimate $\hat{x}(k \mid k; n)$ of the n-th EKF. This approach has, among others, the advantage of producing estimates of both the number of sources and their DOA in real time; i.e. there is no need to collect a large set of measurements. In addition, if the number of sources changes, we would expect the a posteriori probabilities to reflect this change and select the correct filter, again in real time.

4. SIMULATION RESULTS

A large number of simulation experiments have been carried out in order to compare the performance of three DOA estimation algorithms under various operating conditions. The algorithms simulated are:

a) the conventional beamforming algorithm (CBF) [10], which is one of the first approaches to be employed in the DOA problem;

b) the widely used Multiple Signal Classification (MUSIC) algorithm [11];

c) the Multi-Model Partitioning Filter (MMPF) proposed in [12].

Our objective is to test the above algorithms in the following adverse cases, or combinations thereof:

- presence of multiple sources
- small number of available measurements (snapshots)
- small number of operational sensors

In all cases, an array of isotropic sensors was used, with equal sensor spacing of $d = 0.45\lambda$. The number m of available sensors varied between the extremes of 2 and 15. In each case, a constant number of sources from 1 to 5 was simulated. In order to be consistent, the sources were uniformly spaced at 10^o apart. The number of snapshots used by the algorithms varied from 5 to 25. Finally, the received signal-to-noise ratio was kept at a favourable 15 dB in all experiments.

The MMPF was implemented using a bank of six EKFs, each of which assumes a different number of sources, from 1 to $n_{MAX} = 6$, being present in the received signal. It must be noted that the algorithm performs simultaneously both tasks of identifying the number of sources, and estimating the corresponding directions. In contrast, the CBF and MUSIC algorithms rely on other system identification techniques [8], [9] to obtain an estimate of the number of sources, before DOA estimation can be carried out. In order to focus on the performance of the estimators without introducing unnecessary complications, we chose to feed the correct value of the number of sources to these algorithms. The MMPF, on the other hand, was fully implemented and obtained its own estimates of the number of sources via Eqs. (7) and (8).

Fig. 2 presents collectively the results of several experiments in order to demonstrate the result of fewer sensors on estimator performance. We start with a base case of an array of $m = 15$ sensors, and five sources emitting from angles 10^o, …, 50^o. The results for this case are depicted on the first line (top three graphs) of Fig. 2. The leftmost graph shows the evolution in time of the a posteriori probabilities, which are calculated by the MMPF using Eqs. (7) and (8). It can be seen that, after 8-9 snapshots, the probability for $n = 5$ (i.e. five sources present) has converged to 1. The second graph displays the estimates of the particular EKF within the MMPF bank, which is implemented assuming that $n = 5$. The estimates converge to the true DOA values within about 15 snapshots, with a small steady-state error. Finally, the rightmost graphs shows the estimates obtained by the CBF and MUSIC algorithms, after processing all 25 snapshots. It is apparent that the MUSIC estimates are very accurate, while the CBF estimates are barely adequate.

The performance of all algorithms deteriorates with the availability of fewer sensors. However, the CBF does not produce acceptable results except for the base case. In contrast, the MUSIC algorithm is able to provide accurate DOA estimates until $m = 11$. Below that number of sensors, the algorithm cannot differentiate between different sources, although it is given the correct value of n. The MMPF can still detect the number of sources and provide DOA estimates for the case of only 7 sensors available.

In experiments with fewer sources present, a smaller number of sensors were shown to be sufficient for obtaining DOA estimates. Results for the case of 3 sources are shown for comparison in Fig. 3, all other parameters being identical. The minimum number of sensors required for the MUSIC and MMPF algorithms to provide DOA estimates is 6 and 5, respectively. For the case of 2 sensors (not shown) the corresponding values are 4 and 3.

Next we examine the estimates obtained based on a small number of snapshots. Our base case here involves an array of 15 sensors, while 20 snapshots are available. The results are presented in Fig. 4 for the case of five sources present, and Fig. 5 for the case of three sources. In the five source case, the MMPF algorithm requires about 15 snapshots, while MUSIC needs about 20. In the cases of three or fewer sources, all three algorithms perform satisfactorily with as few as 5 snapshots.

It is apparent that, in general, the performance of the DOA estimation algorithms examined depends more critically on the number of available sensors than on the number of snapshots. However, this dependence is more pronounced in the case where several sources are detected. We try to summarize the performance of the MMPF and MUSIC algorithms as a function of both the number of available sensors and the number of snapshots processed in Figs. 6 and 7. These figures give a graphic presentation of our experimental results for the borderline cases of 5 and 2 sources, and for all combinations of up to 25 snapshots and 15 sensors. Black cells show sensor/snapshot value combinations where the corresponding algorithm cannot operate, or does not produce any valid results; white cells indicate detection of the correct number of sources and estimation of the DOAs; while different shades of grey specify partial source detection/estimation. In all cases the performance degrades from the upper right corner (all sources detected and correctly estimated) to the lower left corner (no results at all).

The results presented indicate that the MUSIC algorithm is heavily affected by the combination of a small number of sensors and several sources present. The MMPF can tolerate fewer sensors regardless of the number of sources, as long as the latter is taken into account in the filter bank. The situation is reversed when just two sources are present, where the performance of MUSIC dramatically improves.

5. CONCLUSIONS

In this paper we have presented the results of a large number of simulation experiments in order to compare the performance of the MMPF in DOA estimation to two well established algorithms, namely beamforming and MUSIC. Our focus was in linking the performance of these algorithms to adverse situations including availability of a small number of sensors, presence of several sources, and small number of snapshots. Our results indicate that the performance of the conventional beamforming algorithm is heavily degraded in the cases of either fewer sensors or several sources. On the other hand, MUSIC operates satisfactorily, except for the combination of fewer sensors and several sources. The MMPF is least affected by these parameters; however, it is outperformed by the MUSIC algorithm in the presence of one or a few sources. Finally, the performance of all algorithms is less heavily dependant on the number of snapshots taken.

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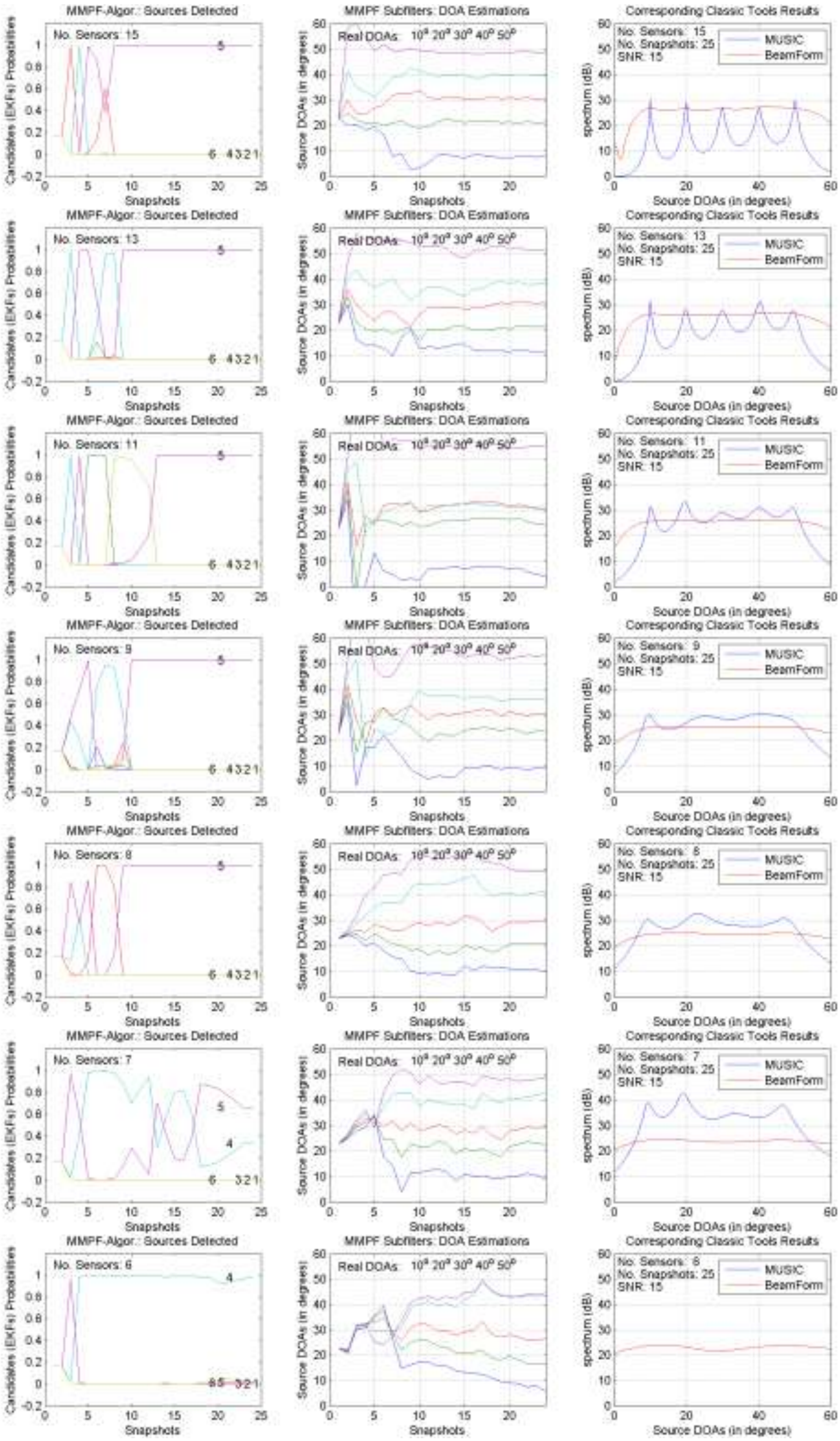

Fig. 2. Simulation results with decreasing number of available sensors (top to bottom). Five sources present in all cases. Leftmost graphs: number of sources detected by the MMPF. Central graphs: MMPF DOA estimates. Rightmost graphs: DOA estimates by the CBF and MUSIC algorithms.

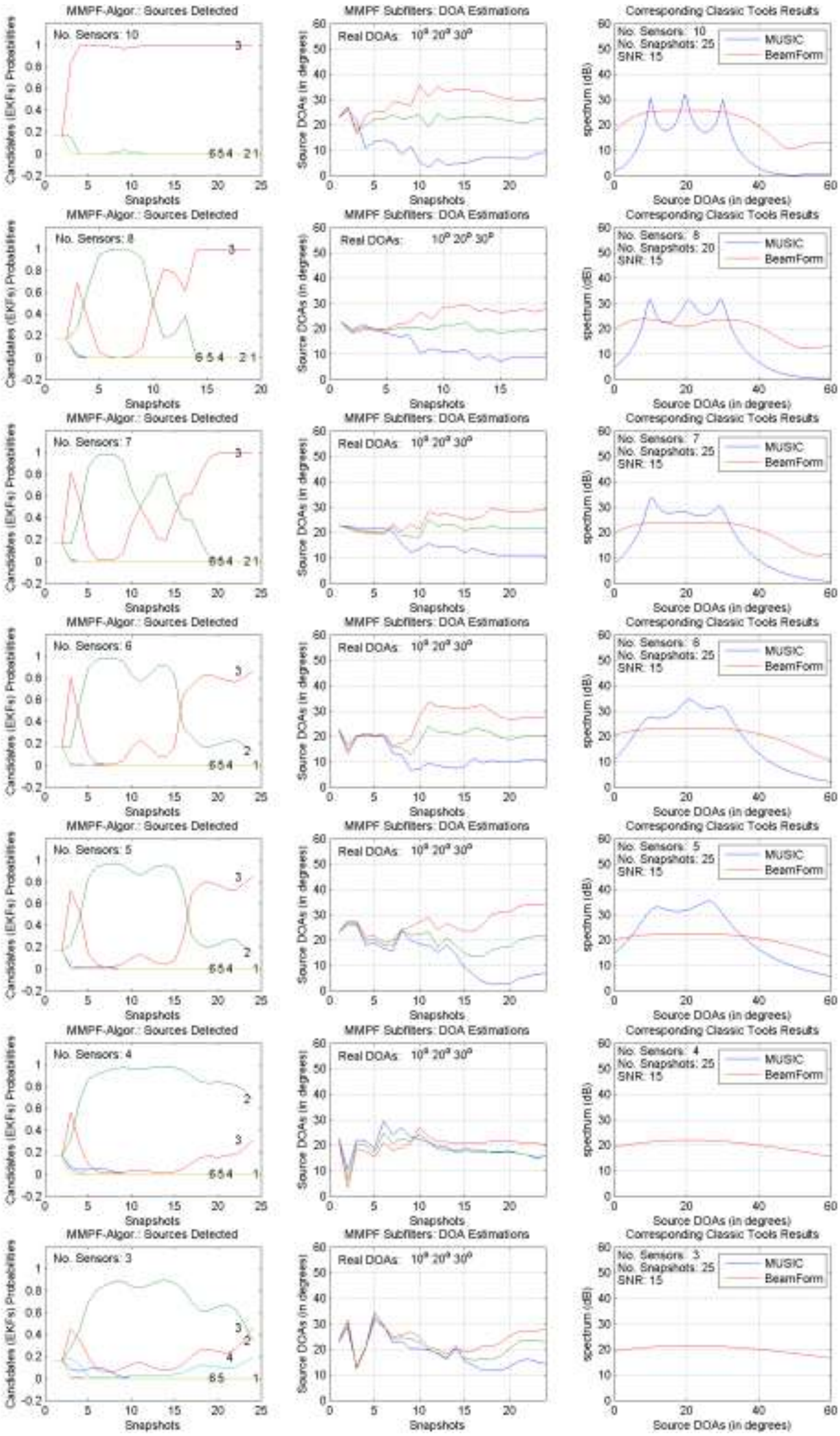

Fig. 3. Simulation results with decreasing number of available sensors (top to bottom). Three sources present in all cases.

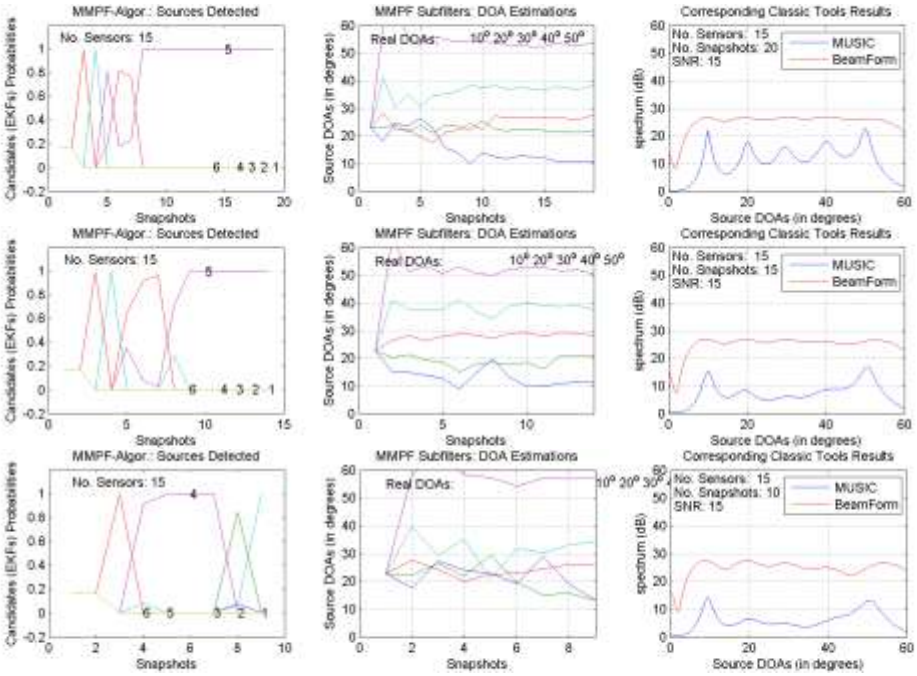

Fig. 4. Simulation results with decreasing number of snapshots (top to bottom). Five sources present.

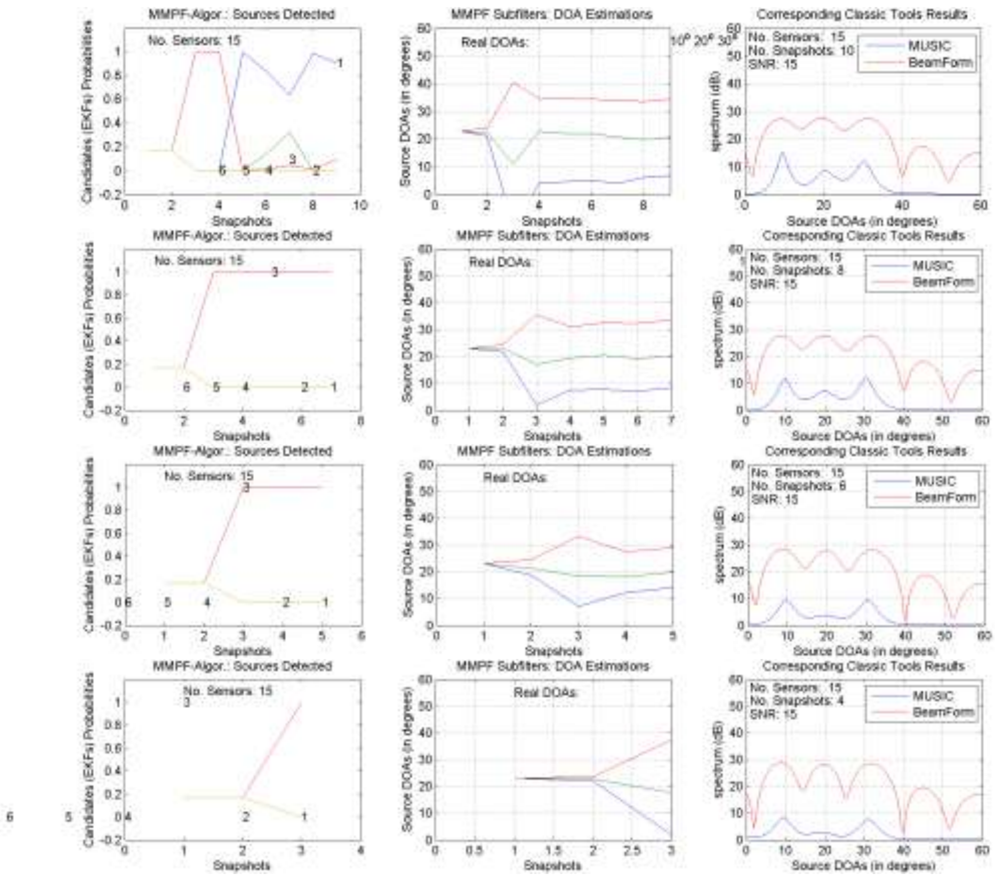

Fig. 5. Simulation results with decreasing number of snapshots (top to bottom). Three sources present.

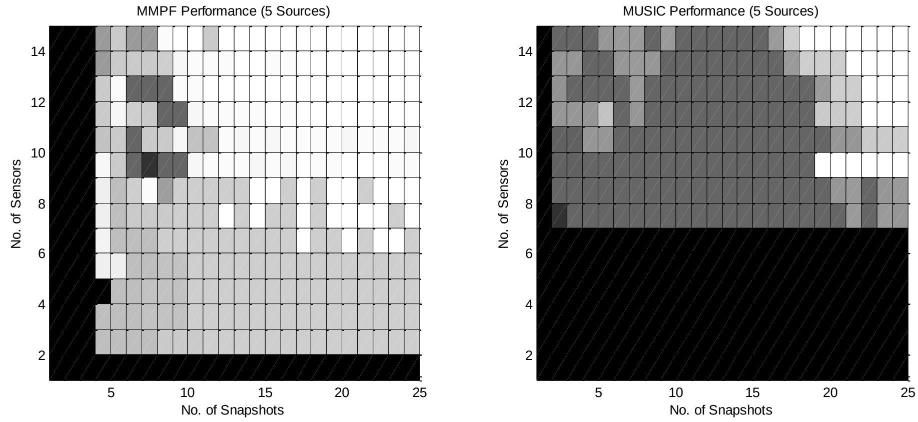

Fig. 6. Performance of the MMPF and MUSIC as a function of the number of available sensors and snapshots. Five sources present. Black cells: no detection/estimation. Dark grey: 2 or 3 sources detected, and 3 or less estimated correctly. Light grey: 4 or 5 detected, and 4 or less estimated correctly. White: 5 out of 5 detected and estimated.

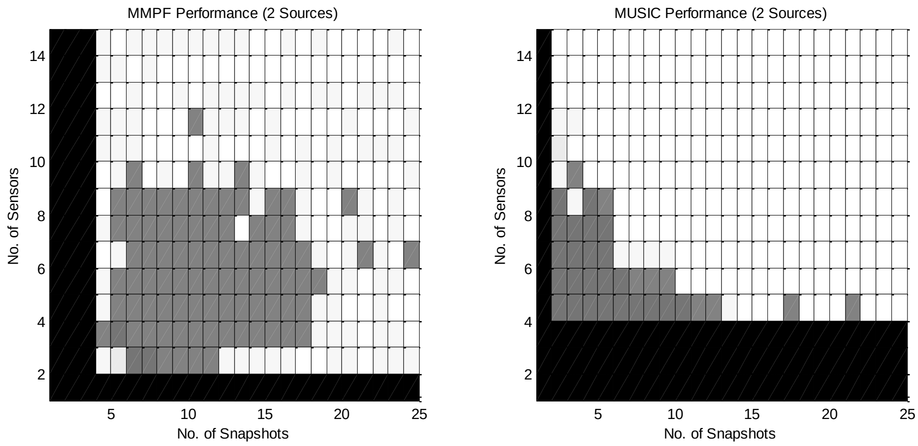

Fig. 7. Performance of the MMPF and MUSIC in the case of two sources present. Black cells: no detection/estimation. Dark grey: 1 source detected and estimated correctly. Light grey: 2 sources detected, 1 estimated correctly. White: 2 out of 2 detected and estimated.