Structural Reliability Improvement using Non-Linear and Adaptive Multi-Model Techniques

VASSILIOS C. MOUSSAS
Department of Civil Infrastructure Works
School of Technological Applications (S.T.EF.), Tech. Educ. Inst. (T.E.I.) of Athens
Ag. Spiridona Str., Egaleo-Athens GR-12210
GREECE
vmouss@teiath.gr, http://www.vmoussas.com

Abstract: - Structural reliability of a complex structure is related to the residual lifetime of its components. Structural components often contain flaws that propagate due to fatigue and when the crack size becomes critical they eventually fail. In this paper, a number of nonlinear and adaptive identification algorithms are applied to the problem of Fatigue Crack Growth (FCG) monitoring and identification, in order to improve the prediction of the residual time to failure. Several algorithms ranging from simple Non-Linear Least Squares (NLLS) to Extended Kalman Filter (EKF) and adaptive Multi-Model Partitioning algorithms (MMPA) are tested in order to compare their efficiency. As it is shown, using real experimental data, the more advanced identification algorithms have superior performance in estimating future crack size and predicting the residual lifetime of a component.

Key-words: - Fatigue Crack Growth, Failure Prediction, Residual Lifetime, EKF, Adaptive, Multi-Model Partitioning.

1 Introduction

Fatigue crack analysis is an essential tool for life prediction and maintenance of structural components that are subjected to cyclic stresses over a prolonged period of time. Lifetime predictions and in-service inspections of each component are used to update the reliability analysis of the overall structure.

Fatigue crack growth (FCG) monitoring and failure prediction are critical in numerous engineering applications such as civil engineering structures, (e.g. bridges, multi-storey buildings, offshore platforms, etc) [1], space, aircraft and ship applications (e.g. reusable spacecrafts, airplanes, helicopters, etc) [2], [3] complex and high risk plants (e.g. chemical factories, nuclear reactors, etc) [4], and, in general, any rare, expensive, or, dangerous structure that is impossible to test a priori in statistically large samples.

For practical applications it is vitally important to have on-line real-time monitoring and on-line estimation/identification of the FCG, in order to obtain earlier and more accurate predictions of the remaining lifetime to failure. Any effort to attain the above goals requires a realistic mathematical model of FCG and effective algorithms for accurate, fast and efficient monitoring, estimation and accurate prediction of FCG and the residual life-time.

In the past, several models of the mechanism of rupture due to fatigue have been proposed. Although no complete theoretical model exists, there is a large number of semi-empirical models available, of varying realism, complexity and difficulty in their application [5], [6]. All these models are nonlinear equations that follow the Linear Elastic Fracture Mechanics (LEFM) concepts. In order to utilize these models several methods were proposed and used with varying success, such as: Linear Regression (LR, a standard approach for models with a linear logarithmic form) [7], [8], Non-Linear Least Squares (NLLS) and Extended Kalman Filter (EKF) (for nonlinear and more complex models) [9], [10], [11] or even the adaptive Multi-Model Partitioning Algorithms (MMPA) [12], [13].

In this paper, the aforementioned non-lineal or adaptive techniques are compared, in terms of residual lifetime prediction accuracy, using real experimental data. Their performance is tested both in estimating the crack size, as well as, in predicting the residual lifetime to failure.

2 The FCG Models

A large number of FCG models are available in the literature. Although some Markovian [18] or ARMA [19] models have been investigated too, most of the FCG models are semi-empirical deterministic laws of the form: $da/dN = g(a, C, n, \Delta S, ...)$ where, a is the crack length, N the number of fatigue cycles and

$g(a)$ a nonlinear function of the crack size a and the material or loading parameters (e.g. Shanley, Paris, Forman, Larsen-Yang equations, etc.) [5], [6]. In this work we concentrate on simple and widely used laws such as Shanley and Paris. The nonlinear functions for these FCG laws are:

$$da/dN = Ca^n \quad \text{(Shanley)} \tag{1}$$

$$da/dN = C\left[\Delta K(a)\right]^n \quad \text{(Paris)} \tag{2}$$

where, $\Delta K(a) = \Delta S(\pi a)^{1/2} Y$, ΔS is the loading range, Y is a function of the geometry and C & n are the material parameters to be identified.

Both equations can be easily integrated, and, in addition, using a simple transformation, Shanley equation becomes identical to Paris equation for Infinite Plates (Paris-IP). For the above reasons, this form of FCG law (Shanley or Paris-IP), although less accurate, is finally selected in order to compare the capabilities of the identification algorithms, as, our results will be more general and equally applicable to any FCG law.

For Linear Regression (LR) techniques (as most standard approaches) [7], [8], the non-linear FCG equations (1,2) become linear by taking, their logarithmic form (3) and by transforming the raw FCG data respectively, e.g.:

$$\log(da/dN) = \log C + n\log\left(a,\ or,\ \Delta K(a)\right) \tag{3}$$

When applying Non-Linear techniques (NLLS) the non-linear FCG equations (1,2) may be used directly on the raw FCG data [9]. As suggested also by [20], [21], parameter estimation of the FCG laws should be performed using the crack length a as the independent variable instead of N. Therefore, equations (1,2) can be written in a non-linear recursive form, provided that step Δa is sufficiently small:

$$N_{k+1} - N_k = \Delta a / g\left(a_k, C, n, ...\right), \ or, \tag{4}$$

$$N_{k+1} = N_k + f_R\left(a_k, \Delta a_k, C, n, ...\right) \tag{5}$$

As all experimental results show that the crack propagation is a stochastic phenomenon [24], [25], advanced stochastic techniques may improve the FCG prediction [10-12].

In order to describe the stochastic nature of FCG in state-space notation, the above semi-empirical and deterministic FCG laws are enhanced by randomizing their parameters (e.g. C & n), or by adding some uncertainty terms, or both. The proposed in [13] model is a recursive State-Space model of the form:

$$x_{k+1} = f\left[k, x_k\right] + w_k \ \ \& \ \ z_k = h\left[k, x_k\right] + v_k \tag{6}$$

This model is suitable for all advanced algorithms, such as the Extended Kalman Filter (EKF), or the Lainiotis Multi-Model Partitioning Algorithm (MMPA), and it is created by the state-space representation of the general equation (5) and therefore is compatible with all semi-empirical laws of type (1,2) [22], [23].

For the parameter identification problem, let θ be the vector containing all the unknown or varying parameters, then the augmented state of the model will be: $x_\theta(k) = [x_k \mid \theta]^T$, and the model equations become:

$$x_{k+1} = f\left[k, x_\theta(k)\right] + w_k \tag{7}$$

$$z_k = h\left[k, x_\theta(k)\right] + v_k \tag{8}$$

Even if the original model (6) were linear, the augmented model will be nonlinear. The nonlinear state space FCG model shown in (7,8) is flexible and able to include and identify any number of unknown parameters in collaboration with the EKF and MMPA algorithms. The state vector contains the model variables: $x_k = [N \quad a \quad \Delta a]^T$ and by augmenting the state vector with the two unknown parameters, i.e. vector $\theta = [C \quad n]^T$, the state at time k becomes $x_\theta = [N \quad a \quad \Delta a \quad C \quad n]^T$. Finally, by using equation (5) for N [13], the state-space FCG model becomes:

$$\begin{bmatrix} N \\ a \\ \Delta a \\ C \\ n \end{bmatrix}_{k+1} = \begin{bmatrix} N + f_R\left(a, \Delta a, C, n\right) \\ a + \Delta a \\ \Delta a \\ C \\ n \end{bmatrix}_k + \begin{bmatrix} w_N \\ w_a \\ w_{\Delta a} \\ w_C \\ w_n \end{bmatrix}_k \tag{9}$$

$$\left[N\right]_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} N \\ a \\ \Delta a \\ C \\ n \end{bmatrix}_k + \left[v_N\right]_k \tag{10}$$

The state-space model of (9,10) is directly applicable to controlled FCG experiments where Δa, N & a, are either controlled or directly observed. If an

indirect Non-Destructive Testing, Inspection or Evaluation (NDT/NDI/NDE) method is used (e.g. Acoustic Emission, Thermal Emission, Potential Drop, etc.), matrix H must be replaced by the corresponding $h[x_k\ldots]$ non-linear function accordingly.

In this nonlinear model the stochastic nature of FCG is expressed by, the state input vector w, the initial state vector x_0, and, the measurement error vector v. Vectors x_0, w & v are Gaussian random variables with variances p_0, Q & R respectively [13].

3 The Identification and Prediction Algorithms

Standard approaches use Linear Regression (LR) techniques but they are restricted to FCG laws having the linear logarithmic form of equation (3). For comparison reasons the selected equations (1, 2) are of this type. Standard LR techniques perform nonlinear transformations (log) of the raw FCG data and provide us with the mean and the distribution of only two parameters: C & n.

By applying the more flexible Non-Linear Least Squares (NLLS) techniques, on the other hand, any FCG law can be used. The model must be written in the recursive form of equation (5) that makes it suitable for the recursive non-linear techniques proposed initially by Marquardt [14]. NLLS techniques can use the raw FCG data without transformation and they provide us with the mean and variance of any unknown parameter of the FCG model.

3.1 The EKF algorithm

The EKF is a technique that applies the linear optimal Kalman Filter to a linearized version of the non-linear model. Let us consider the nonlinear model of equations (7, 8). By taking the first terms of the Taylor expansion of the nonlinear quantities we have [17]:

$$\begin{aligned} f[k,x(k)] &= f[k,\hat{x}(k/k)] + F(k)\cdot[x(k)-\hat{x}(k/k)]+\ldots \\ h[k,x(k)] &= h[k,\hat{x}(k/k-1)] + H(k)\cdot[x(k)-\hat{x}(k/k-1)]+\ldots \end{aligned} \quad (11)$$

where,

$$F(k)=\left.\frac{\partial f(k,x)}{\partial x}\right|_{x=\hat{x}(k/k)},\ H(k)=\left.\frac{\partial h(k,x)}{\partial x}\right|_{x=\hat{x}(k/k-1)} \quad (12)$$

A linear approximation of the original model is then given by:

$$\begin{aligned} x(k+1) &= F(k)\cdot x(k)+G(k)\cdot w(k)+a(k) \\ z(k) &= H(k)\cdot x(k)+v(k)+b(k) \end{aligned} \quad (13)$$

where,

$$\begin{aligned} a(k) &= f\left[k,\hat{x}(k/k)\right]-F(k)\hat{x}(k/k) \quad \& \\ b(k) &= h\left[k,\hat{x}(k/k-1)\right]-H(k)\hat{x}(k/k-1) \end{aligned} \quad (14)$$

The 1st order EKF is a Kalman Filter applied on model (13) with the quantities of eq. (12) recalculated at each iteration.

The state augmentation in EKF is necessary for parameter identification, but presents some difficulties to the applied algorithms. When dealing with augmented models, the different (higher) dimensionality can create some problems for EKF that should be investigated. The model matrices are larger and more complex with higher computational requirements. A linear or trivially nonlinear model becomes highly nonlinear. More reasons for divergence appear and the robustness of the algorithm becomes questionable. Extensive tests, with Monte Carlo runs and various conditions are required to successfully tune the algorithm to the investigated problem.

3.2 The Multi-Model Partitioning Algorithm

The MMPA technique bypasses the divergence and nonlinearity problems of the augmented EKF by using more than one candidate models in parallel.

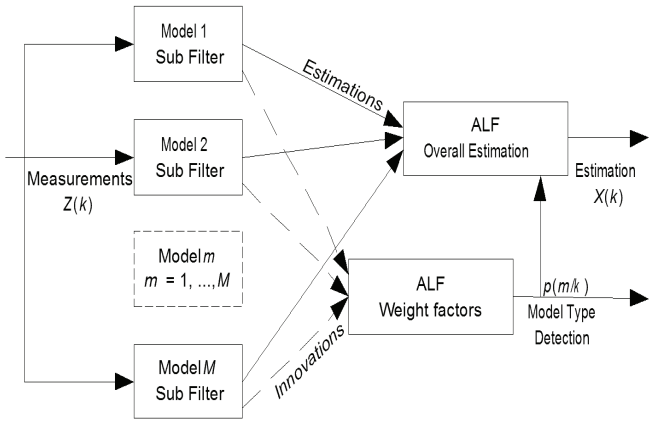

Fig. 1: Sub-filter parallel structure of the adaptive Lainiotis Multi-Model Partitioning Algorithm (MMPA).

Let us consider the model with unknown parameters shown in eq. (8), but, without augmenting the state by the vector θ.

$$\begin{aligned} x(k+1) &= f\left[k,x(k);\theta\right]+g\left[k,x(k)\right]\cdot w(k) \\ z(k) &= h\left[k,x(k);\theta\right]+v(k) \end{aligned} \quad (15)$$

In the above model, θ is a random variable with known a-priori probability density $p(\theta/0)$. Given a measurement set $Z_k = \{z(1), z(2), ..., z(k)\}$, the optimal estimation $\hat{x}(k/k)$ and its variance $P(k/k)$ are [15-16]:

$$\hat{x}(k/k) = \sum_{i=1}^{M} \hat{x}_i(k/k)\, p(\theta_i/k)$$

$$P(k/k) = \sum_{i=1}^{M} \left[P_i(k/k) + \left\| \hat{x}(k/k) - \hat{x}_i(k/k) \right\|^2 \right] \times \quad (16)$$
$$\times\, p(\theta_i/k)$$

where, index i indicates the quantity corresponding to the value θ_i of array θ, where, $\hat{x}_i(k/k)$ and $P_i(k/k)$ can be calculated using a Kalman or EKF sub-filter designed for the model with parameter θ_i. The posteriori probability density $p(\theta_i/k)$ of θ_i, given the measurements Z_k is:

$$p(\theta_i/k) = \frac{L_i(k/k)}{\sum_{j=1}^{M} L_j(k/k) \cdot p(\theta_j/k-1)} \cdot p(\theta_i/k-1) \quad (17)$$

$$L_i(k/k) = \left| P_{\tilde{z}_i}(k/k-1) \right|^{-1/2} \mathrm{e}^{-\frac{1}{2}\left\| \tilde{z}_i(k/k-1) \right\|^2 \cdot P_{\tilde{z}_i}^{-1}(k/k-1)} \quad (18)$$

The MMPA bypasses the divergence problems of the EKF by using more than one candidate models in parallel. When a sub-filter deviates it is isolated by zeroing its posteriori probability density $p(\theta_i/k)$ and the final result is a weighted sum of the surviving sub-filters.

4. Comparison using Experimental Data

The experimental results of Virkler, Hillberry & Goel [24], shown in Fig. 2, have been selected for consideration.

The specimens used to obtain these results were centre cracked panels of 2024-T3 aluminum alloy 2.54 mm thick, 558.8 mm long and 152.4 mm wide. The total number of specimens was 68. The load was of sinusoidal form with frequency 20 Hz, maximum value P_{max} = 23.353 kN and load ratio R = 0.20. Data recording started at crack length, a, of 9 mm and extended to a final length of 49.80 mm. The accumulated number of cycles were recorded for each Δa = 0.20 mm first, and it was increased to 0.40 and to 0.80 mm after the crack lengths of 36.20 mm and 44.20 mm respectively. Thus each specimen produced 165 data points.

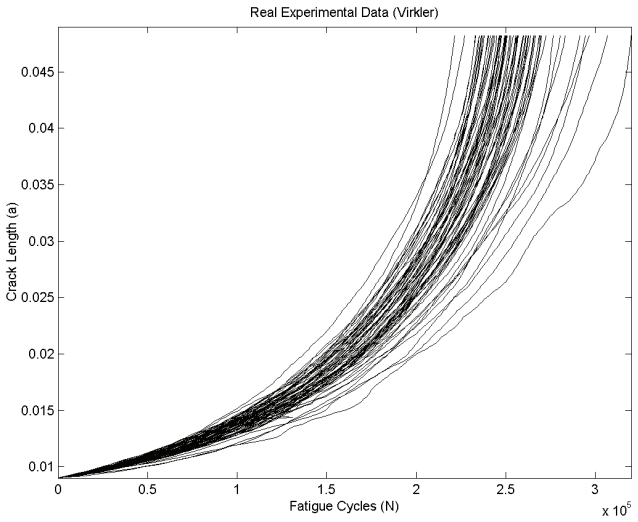

Fig. 2: Crack growth histories (a vs N) from 68 specimens.

The experimental curves were first treated using the simpler LR and NLLS methods that produced some average estimation of the model parameters C & n. The mean parameter values for the entire set of the 68 curves, as well as, for each curve separately were estimated (Table 1). These estimations are also used later as initial or reference values.

Based on these distributions one can also create a simulated data set. The simulations, as shown in Fig. 3, represent well the stochastic nature of FCG although they are smooth and do not cross each other.

Depending on the selected FCG law and the unknown parameters we can produce more sets of simulated data (with varying accuracy), but, in this work we focus on the algorithms and their capabilities, so we considered only the simple Shanley (Paris-IP) law [23].

Table 1: Mean parameter values estimated by the LR and NLLS methods

Method	Param.	Mean	Variance
LR	n	1.86009	-
(Shanley)	C	0.00021132	3.29E-09
NLLS	n	1.8091	1.32E-02
(Shanley)	C	0.00016176	1.16E-10

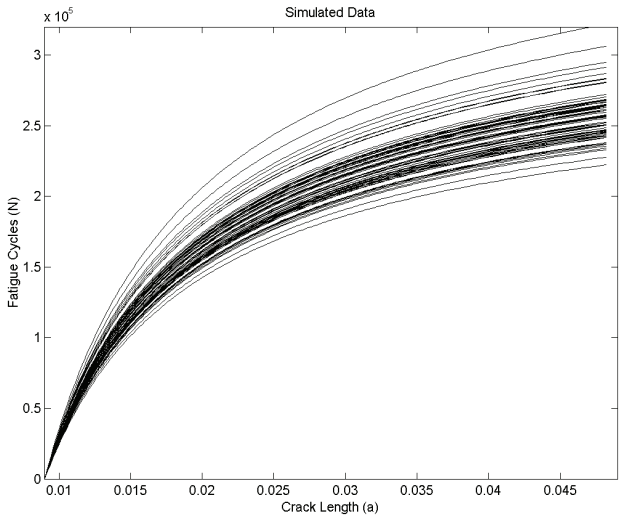

Fig. 3: Simulated data sets (N vs a) recreated using Shanley law and parameter C values from Table 1.

In on-line real time cases, reception and treating of additional data, improves the final predictions. When starting early with fewer points our predictions are not so accurate. After a certain point, the final predictions become more accurate and get closer to the real values. Therefore, depending on the starting point along the crack propagation curve, the prediction of the final crack size and its confidence (variance) vary significantly.

For comparative analysis, the final crack is repeatedly predicted at 3 prediction points that correspond approximately to the 5%, 20% and 50% of a curves data (Fig. 4).

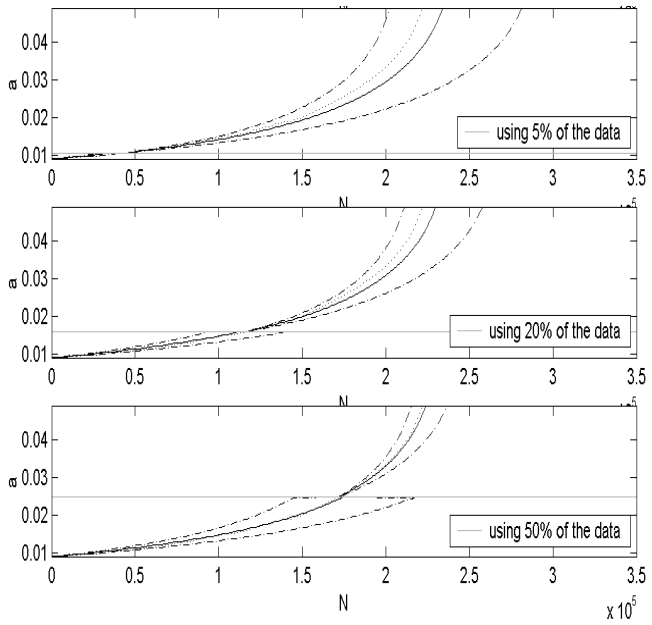

Fig. 4: Simulation of the leftmost curve and prediction of the final crack size from 3 different starting points: the 9th, 36th and 80th point of the curve.

For a better presentation the three separated plots of Fig. 4 are combined in one continuous plot in Fig. 5 that includes the final prediction from every data point of the curve. Fig. 5 presents the results for the leftmost and rightmost curves and shows clearly how the final predictions are improved as more data points become available. The vertical lines represent the amount of data used (5%, 20% & 50%).

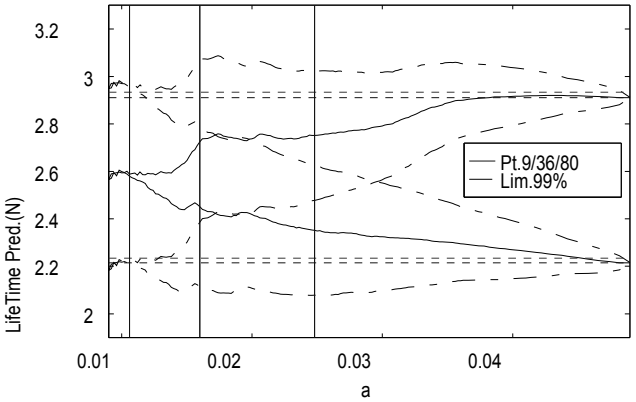

Fig. 5: Continuous prediction of the lifetime for the leftmost & rightmost curves. The vertical lines correspond to the 9th, 36th and 80th point of the curve.

When using the simpler LR and NLLS methods the prediction is based on the apriori known parameter values and distributions.

In Fig. 6 the EKF predictions are presented for the same experiments. The vertical lines again represent the amount of data used (5%, 20% & 50%). Prediction errors start again from the a-priori error and gradually converge to 0, i.e. the correct number of fatigue cycles. As it becomes clear from Fig. 6, the EKF predictions converge much faster to the correct ones.

The main difference and advantage of the EKF technique is that the model parameters are updated on-line when a data point is received [23]. This is also shown in Fig. 6 (lower plot) where parameter C starts from the overall mean value and gradually is updated (estimated) approaching the correct value for each case.

The adaptive estimator MMPA (or ALF) shown in equations (15-16) is finally applied using the nonlinear state-space model of eq. (9-10). The algorithm, instead of state augmentation, uses a number of EKF estimators tuned to the corresponding nonlinear models. Each estimator uses the nonlinear model with a different value for the C parameter. All possible values are within the area defined by the results in Table 1.

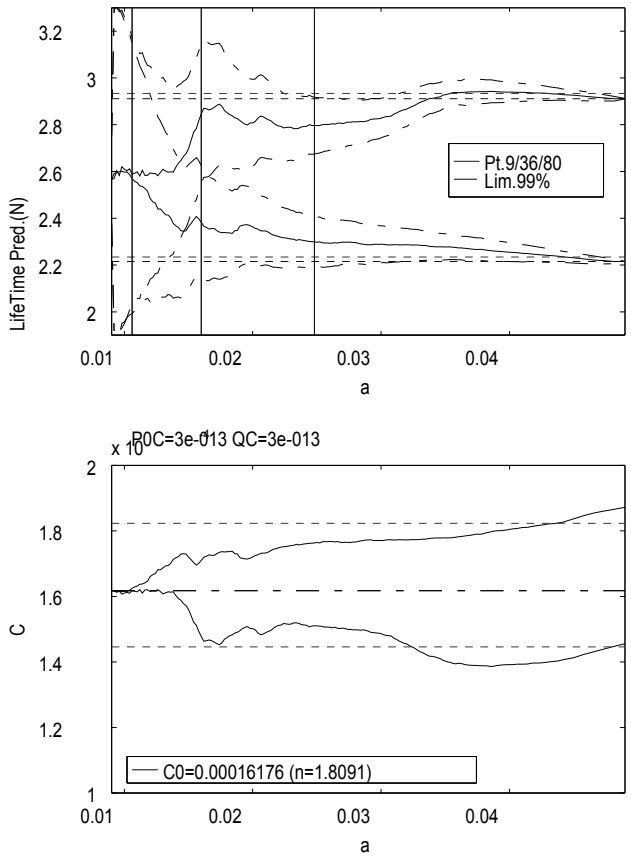

Fig. 6: EKF Lifetime prediction and parameter C estimation.

As it is clearly shown in fig. 7, the MMPA technique reaches the 2% accuracy threshold much earlier, having received only 30% of the dataset, when EKF needs more than 60% of the dataset to reach the same accuracy for lifetime prediction[13].

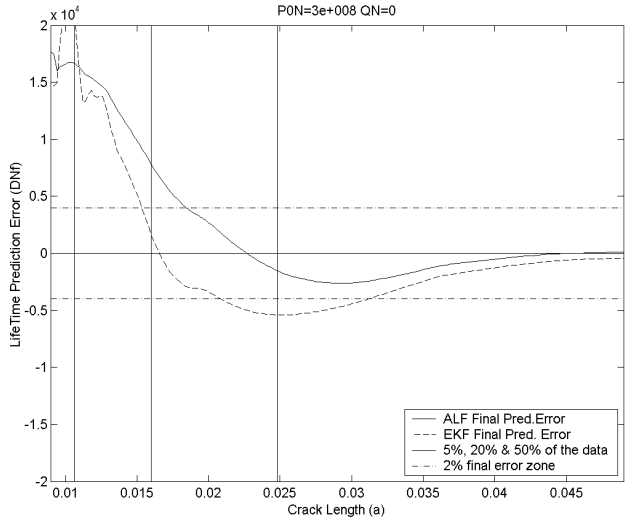

Fig. 7: EKF & MMPA (ALF) lifetime prediction errors.

The MMPA results depend on the number of sub-filters used. The optimal number of these sub-filters depends on the specific case under consideration. In the FCG case, excellent results were obtained in [13] using just 5 to 7 nonlinear filters (fig. 8).

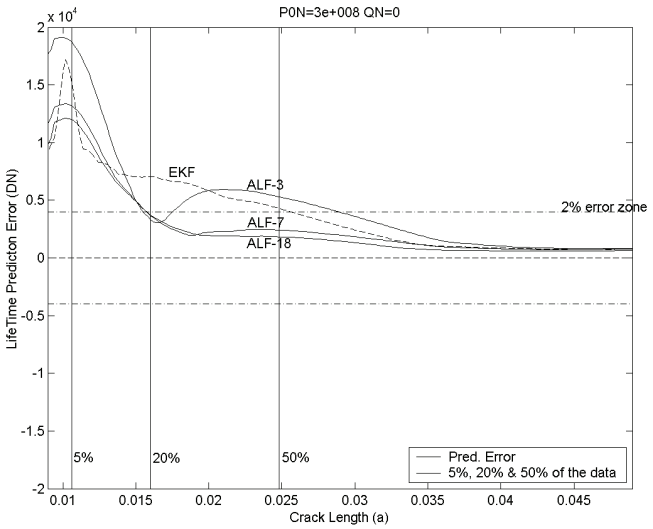

Fig. 8: EKF & MMPA (ALF) lifetime prediction errors. Average results over 10 experiments.

5 Conclusions

A number of nonlinear and adaptive identification algorithms were applied to the problem of Fatigue Crack Growth (FCG) monitoring and identification, in order to improve the prediction of the residual time to failure. Each one of the candidate methods has been already presented in detail in previous works [9],[13],[22-23], using simulated and real data. In this paper the four approaches namely, the standard LR, the Non-Linear Least Squares (NLLS), the Extended Kalman Filter (EKF) and the adaptive Multi-Model Partitioning algorithm (MMPA), were compared using the same sets of data in order to compare their lifetime prediction accuracy. As it is shown, using real experimental data, the more advanced identification algorithms have superior performance in estimating future crack size and predicting the residual lifetime of a component. The results show that all predictors eventually do converge to the actual time to live. However, the MMPA can do it more accurately and much sooner than the EKF or the simpler approaches. More precisely, MMPA converges sooner to the correct lifetime prediction requiring 50% fewer measurements, and leaving more (2x) time for reaction than all other techniques. Further issues, regarding lifetime prediction using much smaller or sparse datasets obtained through in-service NDI, are currently under investigation.

References:

1. Madsen, H. O.; Skjong, R. K.; Kirkemo, F., Probabilistic FCG Analysis of Offshore Structures, with Reliability Updating through Inspection. *In Proceedings of Marine Structural Reliability Symposium, Arlington* VA, Oct 5-6 1987; *Society of Naval Architects and Marine Engineers.*
2. Sutharshana, S.; Creager, M.; Ebbeler, D.; Moore, N., A Probabilistic Fracture Mechanics Approach for Structural Reliability Assessment of Space Flight Systems. In *Advances in fatigue lifetime predictive techniques; Proceedings of the Symposium, San Francisco, CA, Apr 24, 1990 (A92-45226 19-39). Philadelphia, PA, American Society for Testing and Materials*, 1992, p. 234-246.
3. Newman, J. C. Jr., A crack closure model for predicting fatigue crack growth under aircraft spectrum loading. *ASTM STP* 1981, 748, 53-84.
4. Lucia, A. C., Probabilistic Structural Reliability of PWR Pressure Vessels. *Nucl. Eng. Design*, 1985, 87, 35-49.
5. Schutz, W., The Prediction of Fatigue Life in the Crack Initiation and Propagation Stages - A State of the Art Survey, *Eng. Fract. Mech.*, 1979, 11, 405-421.
6. Kaminski, M., On Probabilistic Fatigue Models for Composite Materials, *Int. Journal of Fatigue*, 2002, 24, 477-495.
7. Standard Designation E647-93, Standard test method for measurement of FCG rates. *American Society for Testing Materials ASTM Philadelphia,* PA, 1994, 03.01, 679-706.
8. Fuchs H.O., Stephens R.I., *Metal Fatigue in Engineering*, J. Wiley & Sons, 1980.
9. Moussas, V. C.; Solomos, G.; Lucia, A. C., A General Method for Raw Fatigue Crack Growth Data Processing and Structural Reliability Assessment. In *Computational Stochastic Mechanics, Proceedings of the 2nd International Conference on Computational Stochastic Mechanics, Athens, Greece*, June 12-15, 1994; Spanos, P.D., Ed.; Balkema: Rotterdam, 1994.
10. Ray, A.; Tangirala, S., A Nonlinear Stochastic Model of Fatigue Crack Dynamics. *Prob. Engng. Mech*, 1997, 12 (1), 33-40.
11. Patankar, R.; Ray, A., State-space Modeling of Fatigue Crack Growth in Ductile Alloys,. *Engineering Fracture Mechanics*, 2000, 66, 129-151.
12. Lainiotis, D. G. Adaptive Dynamic Modeling, Risk Assessment, Failure Detection and Correction. In Advances in Structural Reliability, *Proceedings of the Advanced Seminar on Structural Reliability, JRC-Ispra* (VA) Italy, 4-8 Jun 1984; Lucia, A. C., Ed.; D. Reidel Pub Co: Dordrecht, 1984;
13. Moussas V.C., Katsikas S.K., Lainiotis D.G., Adaptive Estimation of FCG using Nonlinear State Space Models, *Stochastic Analysis and Applications*, Vol. 23, 2005, pp.1-18.
14. Marquardt, D. W., An algorithm for least-squares estimation of non-linear parameters, *J. Soc. ind. appl. math.*, Vol.11, 1963, pp.431-441.
15. Lainiotis, D. G., Partitioned Estimation Algorithms, I: Nonlinear estimation, *Information Sciences* 1974, 7, 203-235.
16. Lainiotis, D. G., Partitioning: A Unifying Framework for Adaptive Systems, I: Estimation, *IEEE Proceedings* 1976, 64 (8), 1126-1143.
17. Anderson, B. D. O.; Moore, J. B., *Optimal Filtering*; Prentice-Hall, 1979.
18. Bogdanoff, J. L.; Kozin, F., *Probabilistic Models of Cumulative damage*, Wiley: New York, 1985.
19. Solomos, G. P.; Moussas, V. C., A Time Series Approach to Fatigue Crack Propagation, *Structural Safety* 1991, 9, 211-226.
20. Kozin, F.; Bogdanoff, J. L., A Critical Analysis of some Probabilistic Models of Fatigue Crack Growth, *Eng. Fract. Mech.* 1981, 14, 59-89.
21. Ostergaard, D. F.; Hillberry, B. M., Characterization of the Variability in Fatigue Crack Propagation Data. In *Probabilistic Fracture Mechanics and Fatigue Methods: Applications for Structural Design, ASTM STP* 1983, 798, 97-115.
22. Moussas, V. C., Non-Linear Estimation Algorithms in Component Lifetime Prediction for Improved Structural Safety. *PhD Dissertation* 2002, Dept. Comp. Eng., Univ. of Patras, Greece.
23. Moussas, V. C., Reliability Improvement Using Non-Linear Algorithms and Stochastic State-Space FCG Models, *Applied Research Review Journal of TEI of Piraeus,* Vol. VIII, No.2, 2003, pp. 145-163.
24. Virkler, D. A.; Hillberry, B. M.; Goel, P. K., The Statistical Nature of Fatigue Crack Propagation, *Transactions of ASME J. Engng Mater. Technol*. 1979,101, 148-153.
25. Ortiz, K.; Kiremidjian, A., Time Series Analysis of Fatigue Crack Growth Rate Data, *Eng. Fract. Mech.* 1986, 24, 657-675.