A multi-model approach to fatigue crack growth monitoring and prediction

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Abstract: In this paper an efficient multi-model partitioning algorithm (MMPA) for parameter identification, the so-called Adaptive Lainiotis Filter (ALF), is applied to the problem of Fatigue Crack Growth (FCG) monitoring and identification in order to improve the prediction of the final crack or residual time to failure. The MMPA and Extended Kalman Filter (EKF) algorithms are both tested in order to compare their efficiency. Through extensive analysis and simulation it is demonstrated that the MMPA has superior performance both in parameter identification, as well as, in predicting the remaining lifetime to failure. Furthermore it is shown that the MMPA is fast when implemented in a parallel/distributed-processing mode and it is more robust and converges sooner than the augmented EKF.

Keywords: Adaptive Multi-Model Algorithms; EKF; FCG; Failure; Lifetime Prediction.

Reference to this paper should be made as follows:

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1 INTRODUCTION

Fatigue crack analysis is an essential tool for life prediction and maintenance of structural components. Lifetime predictions and in-service inspections of each component are used to update the reliability analysis of the overall structure. Fatigue crack growth (FCG) monitoring and failure prediction are critical in numerous engineering applications especially in any rare, expensive, or, dangerous structure that is impossible to test a priori in statistically large samples (Sutharshana et al., 1990; Madsen et al., 1987; Lucia, 1985).

For practical applications it is vitally important to have on-line, real-time monitoring and on-line estimation/ identification of the FCG, in order to obtain earlier and more accurate predictions of remaining lifetime to failure. In the past, several models of the mechanism of rupture due to fatigue have been proposed (Schutz, 1979; Kaminski, 2002). Although some Markovian and Time-Series models exist (Solomos and Moussas, 1991; Ortiz and Kiremidjian, 1986; Kozin and Bogdanoff, 1981), most of these models are nonlinear functions that follow the Linear Elastic Fracture Mechanics (LEFM) concepts. To utilize these models several methods were proposed and used with varying success, such as Linear Regression, Non-Linear Least Squares, or, Extended Kalman Filter (EKF) (Ostergaard and Hillberry, 1983; Patankar and Ray, 2000; Moussas et al., 1994).

In this paper we investigate the capabilities of the Multi-Model Partitioning Algorithm (MMPA) proposed by Lainiotis (Lainiotis, 1971), to identify the correct model of the FCG and predict with greater accuracy the crack propagation. The MMPA performance is tested both in FCG estimation, as well as, in predicting the residual lifetime to failure. Furthermore, it is also investigated the robust-ness, the speed and, the parallel/distributed implementation of the algorithm.

2 STATE-SPACE FCG MODEL

Most of the FCG models available in the literature are semi-empirical deterministic laws of the form:

$$da/dN = f_D\left(a,\, C,\, n,\, \Delta S,\, ...\right) \tag{1}$$

where, a is the crack length, N is the number of fatigue cycles, Y and ΔS are material and loading parameters, and $f_D(.)$ is a function of the form:

$$Ca^n\ (Shanley),\ or,\ \ C\left[\Delta S\left(\pi a\right)^{½} Y\right]^n\ (Paris) \tag{2}$$

or, a more elaborated form such as, Forman or Larsen-Yang and other FCG laws (Schutz, 1979; Kaminski, 2002).

Equations (1) and (2) are usually written in a recursive form, provided that step Δa is sufficiently small:

$$N_{k+1} = N_k + f_R\left(a_k,\, \Delta a_k,\, C,\, n,\, ...\right) \tag{3}$$

In order to make the FCG models suitable for the algorithms under consideration, such as MMPA and EKF, they are rewritten in the following recursive State-Space form (Moussas et al., 2005):

$$\begin{bmatrix} N \\ a \\ \Delta a \end{bmatrix}_{k+1} = \begin{bmatrix} N + f_R\left(a, \Delta a, C, n, ...\right) \\ a + \Delta a \\ \Delta a \end{bmatrix}_k + \begin{bmatrix} w_N \\ w_a \\ w_{\Delta a} \end{bmatrix}_k \tag{4}$$

$$\left[N\right]_k = \left[1 \quad 0 \quad 0\right] \cdot \begin{bmatrix} N \\ a \\ \Delta a \end{bmatrix}_k + \left[v_N\right]_k \tag{5}$$

This state-space form of the FCG model is based on the general equation (3). For every semi-empirical law of type (1) and for every set of parameters C and n, a different model implementation is created. By denoting this set of unknown or varying parameters by the vector θ, the general form of the model will be:

$$\begin{aligned} x\left(k+1\right) &= f\left[k, x\left(k\right); \theta\right] + w\left(k\right) \\ z\left(k\right) &= h\left[k, x\left(k\right); \theta\right] + v\left(k\right) \end{aligned} \tag{6}$$

where, the quantities w and v represent the stochastic nature of the FCG.

3 LAINIOTIS' MULTI-MODEL PARTITIONING ALGORITHM

We consider the model with unknown parameters shown in equation (4). In this model, θ is a random variable with known a-priori probability density $p(\theta/0)$. Given the measurement set $Z_k = \{z(1), z(2), \ldots z(k)\}$, the MMPA optimal estimation $\hat{x}(k/k)$ and the variance $P(k/k)$ are (Lainiotis, 1971):

$$\begin{aligned} \hat{x}\left(k/k\right) &= \sum_{i=1}^{M} \hat{x}_i\left(k/k\right) p\left(\theta_i/k\right) \\ P\left(k/k\right) &= \sum_{i=1}^{M} \left[P_i\left(k/k\right) + \left\|\hat{x}\left(k/k\right) - \hat{x}_i\left(k/k\right)\right\|^2\right] p\left(\theta_i/k\right) \end{aligned} \tag{7}$$

where, $\hat{x}_i(k/k)$ and $P_i(k/k)$ can be calculated using a Kalman or Extended Kalman Filter (Anderson and Moore, 1979) designed for each model with parameter θ_i. The a posteriori probability density $p(\theta_i/k)$ of θ_i, given the measurements Z_k is:

$$p\left(\theta_i/k\right) = \frac{L_i\left(k/k\right)}{\sum_{j=1}^{M} L_j\left(k/k\right) \cdot p\left(\theta_j/k-1\right)} \cdot p\left(\theta_i/k-1\right) \tag{8}$$

$$L_i(k/k) = \left|P_{\tilde{z}_i}(k/k-1)\right|^{-\frac{1}{2}} \mathrm{e}^{\left\{-\frac{1}{2}\left\|\tilde{z}_i(k/k-1)\right\|^2 \cdot P_{\tilde{z}_i}^{-1}(k/k-1)\right\}} \tag{9}$$

where, index i indicates the quantity corresponding to the value θ_i of array θ, and, $\tilde{z}_i\left(k/k-1\right)$ and $P_{\tilde{z}_i}\left(k/k-1\right)$ are the conditional innovations and the corresponding covariance matrices produced by the conditional Kalman or Extended Kalman filters:

$$\begin{aligned} P_{\tilde{z}_i}\left(k/k-1\right) &= H\left(k,\theta_i\right) \cdot P\left(k/k-1;\theta_i\right) \cdot H^T\left(k,\theta_i\right) + R\left(k\right) \\ \tilde{z}_i\left(k/k-1\right) &= z\left(k\right) - H\left(k,\theta_i\right) \cdot F\left(k/k-1;\theta_i\right) \cdot \hat{x}\left(k-1/k-1;\theta_i\right) \end{aligned} \tag{10}$$

A schematic description of the above algorithm is shown in Figure 1, where also becomes clear its natural parallel distributed processing architecture.

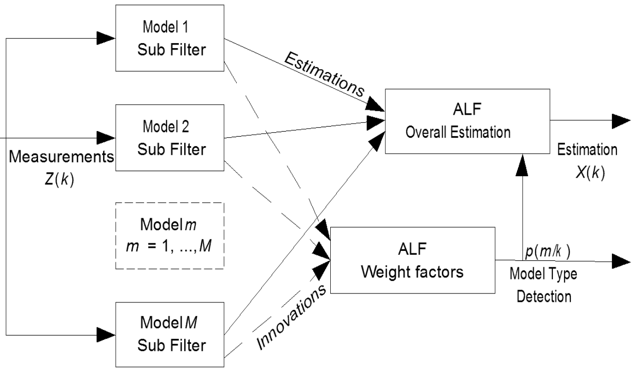

Figure 1 *The MMPA Structure*

4 IDENTIFICATION AND PREDICTION

An experimental data set (Virkler et al., 1979), consisted of 68 *a* vs. *N* curves (Figure 2), is selected for the MMPA tests. Due to the mode of re-cording we consider the crack size *a* as the independent variable and the number of cycles *N* as the dependent variable to predict (number of cycles to failure, or, lifetime). This point of view, i.e. *N* vs. *a* (Figure 3), is in full accord with references (Ortiz and Kiremidjian, 1986; Kozin and Bogdanoff, 1981).

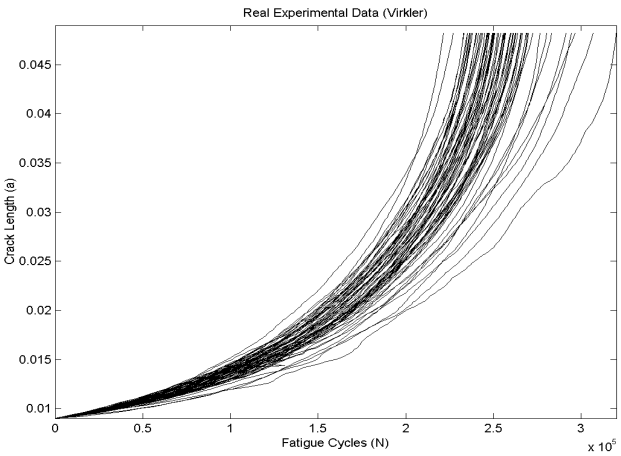

Figure 2 *Virkler et al., FCG Experimental Data (a vs. N).*

We present four (4) cases where MMPA is applied, with varying complexity and efficiency. Their results are shown in Figures 4-7. In our analysis, the final crack is repeatedly predicted at each estimated point, and thus producing a continuous curve indicating the progress of lifetime prediction. For comparison reasons we also use 3 thresholds or prediction points (pts. 9, 36 & 80) that correspond approximately to the 5%, 20% and 50% of a curve's data.

In Case 1, MMPA uses a bank of 3 a priori predictors (no estimation). MMPA selects the correct model using the 60% (Figure 4). In Case 2, MMPA uses 3 estimators (EKF) and using 50% of the data, predicts the correct lifetime even without having the correct model in the bank (Figure 5). In Case 3, MMPA uses 3 identifiers (EKF with augmented state) that estimate the unknown parameters as well. The algorithm converges to the true lifetime predictions using only 20% of the data (Figure 6). Equally efficient is, in Case 4, the MMPA with 7 identifiers (Figure 7).

The MMPA converges always toward the correct model even if this isn't included in the filter bank and also convergences faster than any EKF in the filter bank. The MMPA identifies the un-known parameters and predicts the lifetime using only 20% of the data when the EKF needs over 50%. It also robust to any sub-filter divergence as its corresponding probability is zeroed, and when implemented in parallel processing can be faster than the EKF (Moussas et al., 2005).

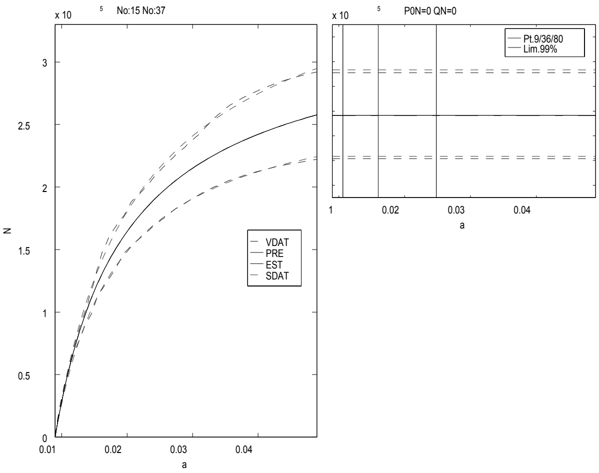

Figure 3 *Real and Simulated FCG curves (N vs. a) – earliest, latest & mean curve (left), and, the corresponding lifetime prediction limits (right).*

5 CONCLUSIONS

An adaptive multi-model partitioning nonlinear algorithm is applied to the FCG problem for model detection and residual lifetime prediction. The MMPA is tested against a classic EKF. Both filters use the state-space representation of a FCG law, and real experimental data. The results show that the MMPA converges to the actual time to live more accurately and much sooner than the EKF, thus requiring fewer measurements, and leaving more time for reaction than the EKF. Due to its partitioned structure, the MMPA is suitable for parallel implementation for increased performance. In addition the MMPA is more robust than a single EKF as it incorporates the mechanism to isolate any diverging sub-filter from the filter bank.

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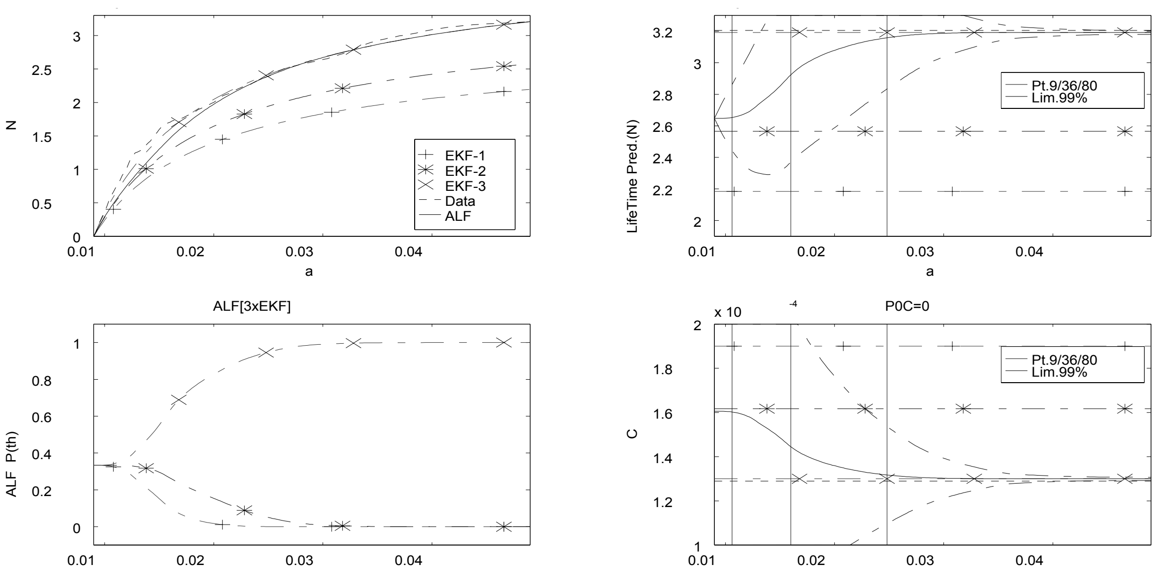

Figure 4 *Case 1: MMPA with 3 Predictors (a priori). - Note for all figures: a) FCG N vs. a curves. b) Continuous lifetime prediction with 99% confidence limits. c) Subfilter’s probability densities. d) Parameter estimation.*

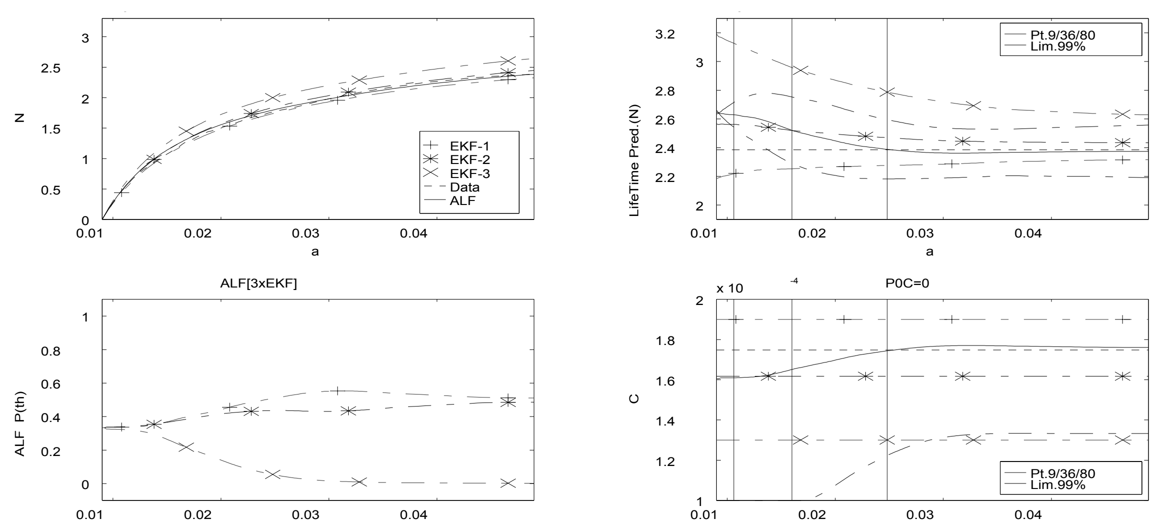

Figure 5 *Case 2: MMPA with 3 Estimators (EKF).*

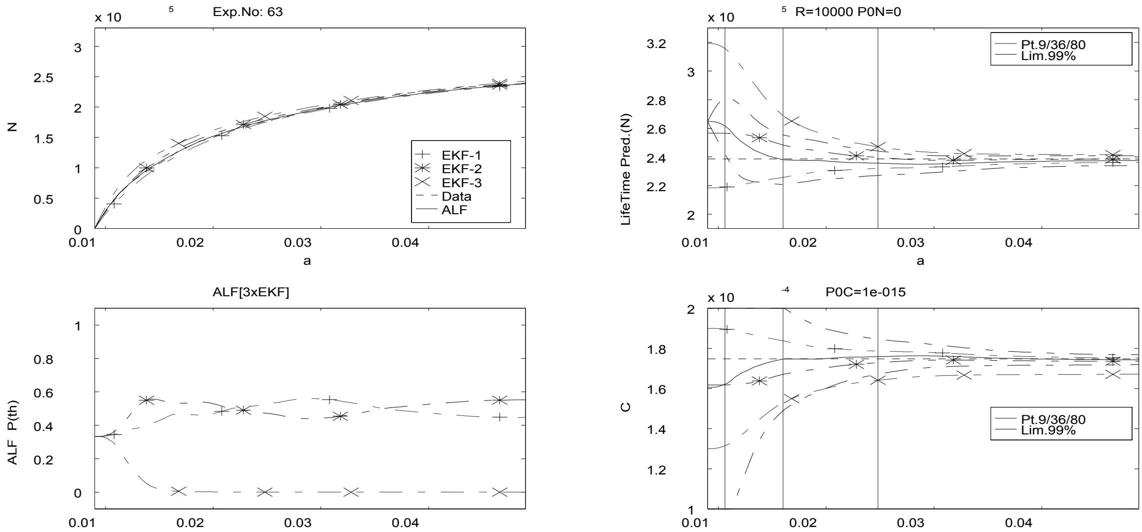

Figure 6 *Case 3: MMPA with 3 Identifiers (EKF-Augm.).*

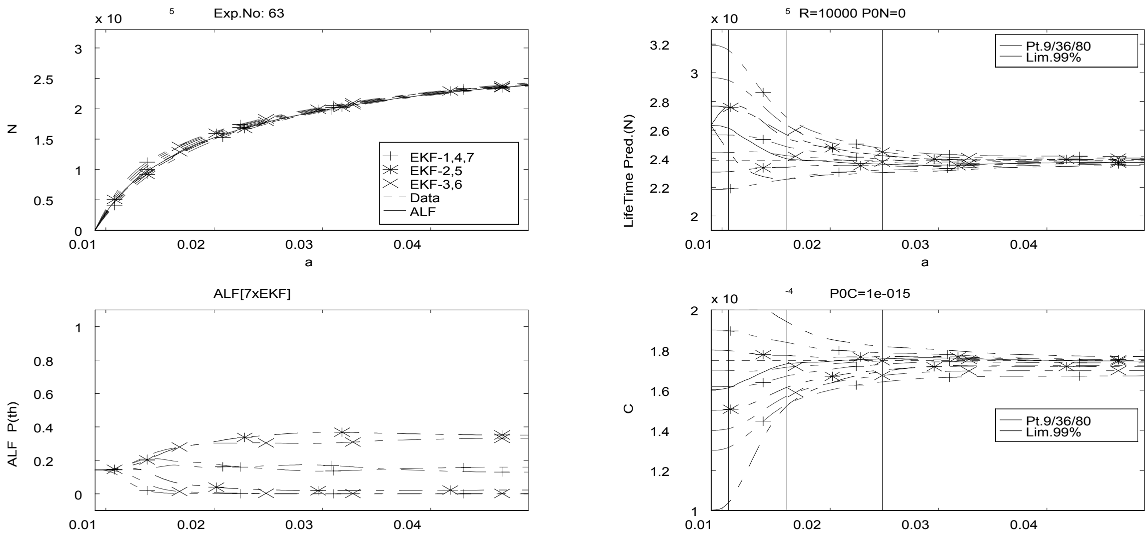

Figure 7 Case 4: MMPA with 7 Identifiers (EKF-Augm.).