ADAPTIVE ON-LINE MULTIPLE SOURCE DETECTION

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ABSTRACT

In this paper an adaptive technique is presented for processing the output of a sensor array, which simultaneously estimates the number of sources and their directions of arrival. The method is based on the reformulation of the problem in the time domain, and the use of the adaptive Multi-Model Partitioning Algorithm (MMPA). The adaptive algorithm identifies the dimensionality of the problem (number of sources) using a bank of Extended Kalman Filters (EKF). The method has the ability of successfully tracking changes in the model structure in real time. This means that, for example, variations in the number of emitting sources are successfully detected. Simulation results demonstrate the performance of the proposed method in multiple source detection and DOA estimation.

1. INTRODUCTION

The problem of direction of arrival (DOA) estimation given a set of measurements of the output of a sensor array has been a topic of considerable interest in the literature. Most of the proposed solutions employ the Maximum Likelihood (ML) approach, which has appeared in two versions, known as the stochastic ML method and the deterministic ML method [1], [2]. Sub-optimal techniques with reduced computational load, such as the Minimum Variance (MV) method [3], the MUSIC method [4], the related Minimum Norm method [5], the ESPRIT estimator [6] and the weighted subspace fitting (WSF) algorithm [7] have become quite popular. Implementations of these techniques have been based on eigenvalue decomposition (ED) of the sample correlation matrix or on singular value decomposition (SVD) of the data matrix. These are either off-line or two-step processing methods and their performance is critically dependent on the validity of their underlying assumptions.

A crucial assumption made when such methods are employed is that the number of sources that generate sine waves or incident plane waves contained in the received signal is known. In many practical situations, however, this prior knowledge may well be unavailable. In these applications, a key issue involved in the development of a suitable model for the received signal is the detection of the number of sources contained in the model. Well known approaches to this problem include the Final Prediction Error (FPE) criterion, Akaike's Information Criterion (AIC) [8] and the Minimum Description Length (MDL) Criterion [9]. Most of the techniques that result from the above criteria suffer from various deficiencies, such as model overfit or underfit, Gaussian assumptions and large sample requirements.

The algorithm proposed herein addresses the combined problem of detecting a time-varying number of emitting sources and estimating their directions of arrival. Our approach consists of transfering the problem to the time domain, and using the Multi-Model Partitioning Algorithm (MMPA) [10] for estimation of the number of sources and Extended Kalman Filters (EKFs) for estimation of the corresponding DOAs.

2. PROBLEM REFORMULATION

Consider an m element array of sensors and n far-field point sources. We define the $m \times 1$ vector $a(\varphi_i)$ to be the complex array response for a source at direction φ_i. Assuming that n signals are simultaneously intercepted, under the narrowband assumption, the array output $z(t)$ is modeled by the following equation:

$$z(t) = A(\varphi)s(t) + v(t) \quad (1)$$

where:

$$z(t) = [z_1(t)\ z_2(t) \dots z_m(t)]^T \quad (2)$$

$$A(\varphi) = [\alpha(\varphi_1)\ \alpha(\varphi_2) \dots \alpha(\varphi_n)]^T \quad (3)$$

$$\varphi = [\varphi_1\ \varphi_2 \dots \varphi_n]^T \quad (4)$$

$$s(t) = [s_1(t)\ s_2(t) \dots s_n(t)]^T \quad (5)$$

The $m \times 1$ vector process $\{v(t)\}$ represents additive measurement noise with variance R. The columns of the $m \times n$ matrix $A(\varphi)$ are the array propagation vectors $a(\varphi_i)$, $i = 1, \dots, n$. These vectors are functions of the directions of arrival and model the array response to a unit waveform from direction φ_i. We assume that the directions of arrival are collected in the parameter vector φ. The vector $s(t)$ comprises the complex signal envelopes received at time t. For the case under consideration, we assume that the signal characteristics are invariant in time, and that the signals emitted by the n sources are:

$$s_i(t) = e^{j\omega_i t}, \quad i = 1,\dots,n \quad (6)$$

The overall steering matrix $A(\varphi)$ can be found by considering the spatial problem of estimating the directions of arrival (DOA) of incident plane waves. Assuming narrowband processing and isotropic sensors, we define the steering matrix $A(\varphi)$:

$$A(\varphi) = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\omega_1\tau_1} & e^{-j\omega_2\tau_2} & \cdots & e^{-j\omega_n\tau_n} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j\omega_1(m-1)\tau_1} & e^{-j\omega_2(m-1)\tau_2} & \cdots & e^{-j\omega_n(m-1)\tau_n} \end{bmatrix} \quad (7)$$

where $\omega_i\tau_i = 2\pi d \sin(\varphi_i)/\lambda_i$, d is the spacing between the sensors, λ is the signal wavelength, ω is the angular frequency and c is the propagation velocity. Assuming, for the moment, that the number of sources is known, the problem is to estimate the spatial frequencies $\omega_i\tau_i$, $i = 1, \dots, n$, based on the measurements $z(t)$. It is then straightforward to compute the directions of arrival signified by the angles of incidence φ_i, $i = 1, \dots, n$. Equation (1) now becomes:

$$z_l(t) = \sum_{i=1}^{n} e^{j\{2\pi ct/\lambda_i - 2\pi(l-1)d\sin(\varphi_i)/\lambda_i\}} + v_l(t) \quad (8)$$

for $l = 1, \dots, m$. We put (8) in a more compact form, $z(t) = h[\lambda_i, \varphi_i, t] + v(t)$, and, by setting the state vector to be $x = [\lambda_1, \lambda_2, \dots, \lambda_n, \varphi_1, \varphi_2, \dots, \varphi_n]^T$, equation (8) becomes:

$$z(t) = h[x(t)] + v(t) \quad (9)$$

The array output is sampled at N distinct time instants, where N the number of measurements, or snapshots. For each snapshot $k = 1, \dots, N$, equation (9) becomes:

$$z_k = h_k(x_k) + v_k \quad (10)$$

This is the nonlinear observation equation of our discrete-time state-space model. The state equation describes the evolution of vector x from $t = t_k$ to $t = t_{k+1}$ and has the form:

$$x_{k+1} = f_k(x_k) + w_k \quad (11)$$

For stationary sources, the state equation (11) is linear [12], but for the general case the state equation must incorporate information such as the sampling interval, the frequency change, the sources bearings, so equation (11) can become more complex or non-linear.

The nonlinear model of (10)-(11) is in the standard state-space form. Therefore, under the assumption that the number of sources n is a known constant, any nonlinear estimation technique, such as the Extended Kalman Filter [13], can be used to process the measurement sequence and obtain a recursive estimate $\hat{x}(k \mid k)$ of the state vector x_k, and in particular of the angles φ_i.

3. THE MULTIMODEL PARTITIONING APPROACH TO SOURCE DETECTION

Let us now relax the assumption that the number of sources n is known; we only know that this number satisfies the condition $n_0 \le n \le n_{MAX}$. It is clear, then, that the correct model describing the measurement process is one of a family of models described by equations (10)-(11), being specified by the actual value of the parameter n. The problem is then to select the correct model among various "*candidate*" models.

Our approach has been to use a parallel bank of EKFs, which operate concurrently on the same measurements. Each filter is based on the system model of (10)-(11), but assumes a different value of the parameter n. At time step k, each filter processes the measurement z_k and produces a state estimate $\hat{x}(k \mid k; n)$ conditioned on the hypothesis that the corresponding value of the parameter n is the correct one.

At a second level, the MMPA uses the output of all filters to select the most likely value of the parameter as the one that maximizes the *a posteriori* probability density $p(n \mid k)$ of the parameter n given the measurement sequence up to and including the k-th snapshot. This density can be calculated recursively [10], [11]:

$$p(n \mid k) = \frac{L(k \mid k; n)}{\sum_{i=n_0}^{n_{MAX}} L(k \mid k; i)\, p(i \mid k-1)}\, p(n \mid k-1) \quad (12)$$

where:

$$\begin{aligned} L(k \mid k; n) &= \left|P_{\tilde{z}}(k \mid k-1; n)\right|^{-1/2} \\ &\times \exp\left\{-\tfrac{1}{2}\tilde{z}^T(k \mid k-1; n) P_{\tilde{z}}^{-1}(k \mid k-1; n)\tilde{z}(k \mid k-1; n)\right\} \end{aligned} \quad (13)$$

and where $\tilde{z}(k \mid k-1; n)$ and $P_{\tilde{z}}(k \mid k-1; n)$ are the conditional innovations and corresponding covariance matrices produced by the conditional EKFs.

At each iteration, the algorithm selects the model that corresponds to the maximum a posteriori probability as the correct one. This probability tends (asymptotically) to one, while the remaining probabilities tend to zero. If the model structure changes, the algorithm senses the variation and increases the corresponding a posteriori probability, while decreasing the remaining ones. Thus the algorithm is adaptive in the sense of being able to track model changes in real time. This procedure incorporates the algorithm's intelligence.

4. SIMULATION RESULTS

The performance of the proposed algorithm has been assessed by simulation. Here we focus on the algorithms capability to detect a variable number of sources and estimate their DOAs simultaneously.

We consider an array of 15 isotropic sensors with equal spacing of d = 0.45λ, and 3 completely coherent signals arriving from directions φ = 10°, 30° and 50°. In our experiment we used 600 snapshots. The number of signals actually received at the array during the experiment varied from 1 to 3 as shown in Table I. A sample of the simulated signal is shown in Figure 1.

TABLE I
SOURCES PRESENT DURING THE EXPERIMENT

Snapshots	Sources
1-100	10°
101-250	10°, 30°
251-450	10°, 30°, 50°
451-600	10°, 30°

For comparison purposes, three conventional algorithms were applied to above time-varying scenario. All algorithms, after having processed the whole batch of 600 snapshots, identified 3 sources, although in fact only 2 were present at the end of the simulation. This result suggests the use of a sliding window approach for these algorithms.

Next, the proposed MMPF algorithm was applied to the same scenario. The bank of Extended Kalman filters consisted of six elements, with the assumed value of the parameter n (number of sources) varying from n_0 = 1 to n_{MAX} = 6. The a posteriori probabilities corresponding to each elemental filter were calculated from (12)-(13). The simulation results are shown in Figure 2.

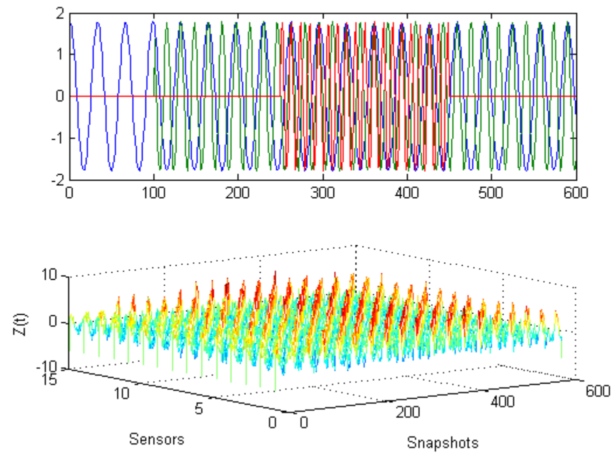

Fig. 1. Emitted signals and sensor array output

During the first part of the experiment (first 100 snapshots), the EKF corresponding to n = 1 produced the a posteriori probability $p(n=1 \mid k) \approx 1$, which indicates a single source, while the EKF estimated the DOA at 10°. The rest EKFs gave erroneous results, which were nevertheless discarded by the algorithm, since the corresponding $p(n \mid k) \approx 0$. During the next part of the experiment the situation reversed, since $p(n=2 \mid k) \approx 1$, indicating two sources. In this case, the second EKF estimates the sources' DOA. In all cases the detection of a change in the number of sources was very fast. Typically 5-10 snapshots were sufficient to determine the correct number of sources. In contrast, conventional algorithms required more snapshots, as shown in Fig. 3.

5. CONCLUSIONS

In this work, the general problem of the DOA estimation has been addressed. A new method for simultaneously estimating the number of sources, as well as the directions of arrival of narrowband signals and the signals emitted by the sources is presented. The problem formulation leads to a nonlinear state-space model with partially unknown structure. Since the number of sources is unknown, a number of different models that possibly fit the data are evaluated, using the Multi-Model Partitioning algorithm. Then, the selection of the correct model is based on the MAP criterion. The method is adaptive, as it is not only able to identify the correct number of sources, but to track changes in the model structure in real time as well. Thus, the method handles also successfully the problem of a variable number of sources. All these desirable characteristics have been verified by simulation experiments. Finally, note that the algorithm exhibits a high degree of parallelism; thus, it can be implemented in a parallel processing environment.

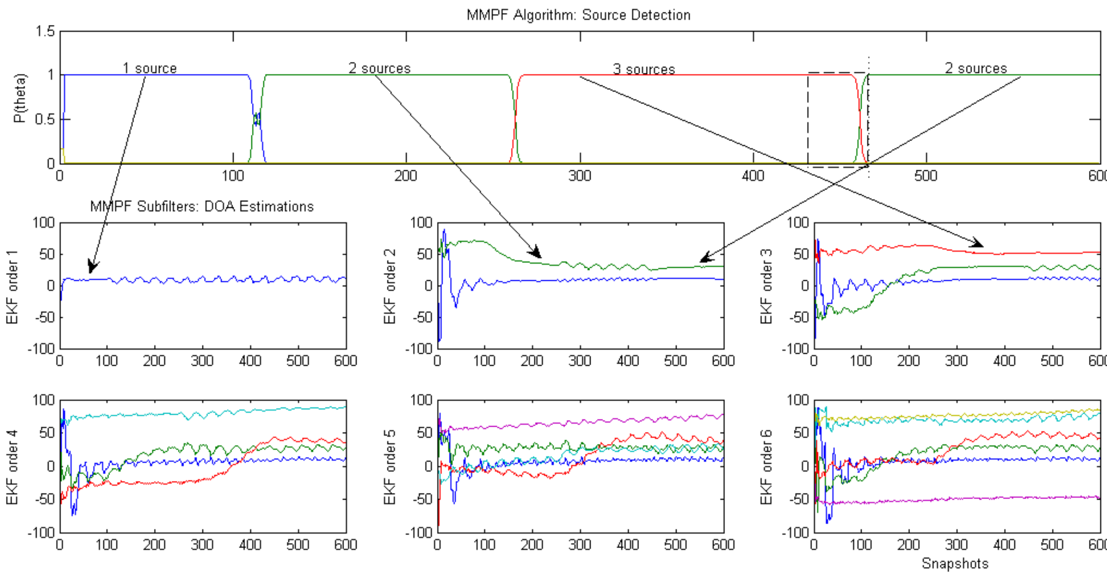

Figure 2. Detection and estimation results for 1-3 sources at directions 10°, 30° and 50°. Dotted lines indicate a sliding window of 40 snapshots used with conventional algorithms.

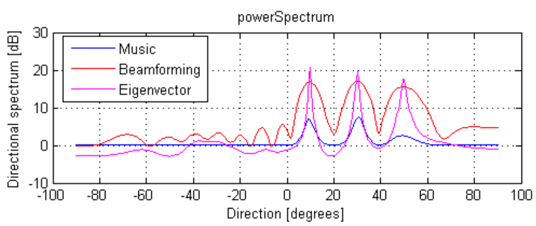

Fig. 3. DOA estimation using conventional methods corresponding to the sliding window of Fig. 2.

6. REFERENCES

[1] M. Pesavento and A.B. Gershman, "Maximum-Likelihood Direction-of-Arrival Estimation in the Presence of Unknown Nonuniform Noise", *IEEE Trans. on Signal Processing*, Vol. 49, No. 7, pp. 1310-134, 2001.

[2] P. Stoica and A. Nehorai, "Performance study of conditional and unconditional direction-of-arrival estimation", *IEEE Trans. Acoust. Speech and Sign. Proc.*, Vol. 38, pp. 1783-1795, 1990.

[3] F.C. Schweppe, "Sensor array data processing for multiple signal sources", *IEEE Trans. Inform. Theory*, pp. 294-305, 1968.

[4] D.H. Jonson, "The application of spectral estimation methods to bearing estimation problems", *Proc. IEEE*, Vol. 70, pp. 1018-1028, 1982.

[5] S. Haykin, "Radar array signal for angle of arrival estimation", in *Array Signal Processing*, S. Haykin (Ed.) Englewood Cliffs, NJ: Prentice-Hall, pp.194-292, 1984.

[6] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques", *IEEE Trans. Acoust., Speech and Signal Processing*, Vol. 31, No. 7, pp. 984-995, 1989.

[7] M. Viberg, B. Ottersten and T. Kailath, "Detection and estimation in sensor arrays using weighted subspace fitting", *IEEE Trans. Signal Processing*, Vol. 39, pp. 2436-2449, 1991.

[8] H. Akaike, "Fitting autoregressive models for prediction", *Ann. Inst. of Stat. Math.,* Vol. 21, pp. 243-347, 1969.

[9] J. Rissanen, "Modeling by shortest data description", *Automatica*, Vol. 14, pp. 465-471, 1978.

[10] D.G. Lainiotis, "Partitioning: A Unifying Framework for Adaptive Systems, I: Estimation", *Proc. of the IEEE*, Vol. 64, pp. 1126-1142, 1976.

[11] K. Watanabe, *Adaptive Estimation and Control: Partitioning Approach*, Prentice-Hall, 1992.

[12] S.D. Likothanassis, S.K. Katsikas and E.N. Demiris, "Direction of arrival estimation via extended Lainiotis filters", in *Proceedings, 5th International Conference on Information Systems, Analysis and Synthesis (ISAS/SCI '99),* Orlando, FL, Vol. VI, pp. 525-530, 1999.

[13] S. Haykin, *Adaptive Filter Theory*, Englewood Cliffs, NJ: Prentice-Hall Int., pp. 445-452, 1991.