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The helicoid multi-groove molecular and the turbomolecular vacuum pumps in molecular state under the scope of statistical behavior of molecules

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In this article we use statistical behavior of molecules to calculate the speed of a gas which is pumped from a helicoid multi-groove molecular pump and from a turbomolecular pump. Thus, we obtain the compression and the pumping speed of the pumps, through very simple formulae, where the main independent variable is the angle between the helixes for the helicoid multi-groove molecular pump and the angle between the blades for the turbomolecular pump. Furthermore we obtain the reconciliation of views of Gaede and Becker from one side and of Kruger and Shapiro from the other, through the geometry of the pumps and their peripheral losses. Copyright © 1996 Elsevier Science Ltd

Introduction

Since 1960 it has been indicated that the compression and the pumping speed of a helicoid multi-groove molecular pump and turbomolecular pump can be expressed as a function of: angular velocity, geometry, nature of the gas and the angle of the helix for the helicoid multi-groove molecular pump and the angle of the blades for the turbomolecular pump.

In other words the attempts, for calculations and design, for such pumps require the determination of the average probable velocity of gas molecules in the pumping direction. There exist two approaches for such a problem. The first approach belongs to Gaede[1] and Becker[2] and according to this, the average velocity of molecules is directly proportional to the angular velocity; this approach is applicable to both pumps. The second approach of Kruger and Shapiro[3] uses the Monte-Carlo method and according to this, the average probable velocity of molecules is proportional to the difference Σ_{12}-Σ_{21} of two probabilities, known as coefficient HO, where Σ_{12} is the probability of molecules to go through a rotational disc with wings in the pumping direction and Σ_{21} is the probability of molecules to go through the same rotational disc but in the opposite direction. The second approach is applied only to the turbomolecular pump.

The aim of the present work is to develop a method which combines the above two approaches in calculating the average probable velocity for both helicoid multi-groove molecular and turbomolecular pumps. Further, by this method, we try to obtain a simpler calculation.

Theory

The gas pumping speed. In our analysis we will consider that the following three statements are valid:

1. On the surface of the pump the molecules collide elastically and the law of cosine is valid.
2. A dynamic equilibrium has been established during the pumping process.
3. At the entry of the pump the pressure of the gas is so small that the probability of collision between any two gas molecules is negligible.

In Figure 1 we see a front part configuration of the helicoid multi-groove molecular vacuum pump. Let M be any common point of the interphase of the pump, between the pumping space and the moving surface. We consider now a plane which includes point M; this plane is normal to the radius of rotation and also parallel to the axis. At the same time M lies on the radius. Any molecule which is emitted from point M with thermal velocity $\vec{V}_m$ will obtain a final velocity equal to $\vec{V}_S = \vec{V}_m + \vec{u}_o$ because the molecule participates in the rotation of the pump, having rotational velocity u_0. The speed projection of $\vec{V}_S$ in the pumping direction (in the direction of the side walls of the opening) MM′ is $V_{S\varphi} = V_S \cdot \cos\lambda$.

In Figure 2 we see a front part configuration of a disc of a turbomolecular pump. Let N be any common point of the interphase between the disc and the pumping space. Let us consider, as we did in Figure 1, a plane which includes point N; this plane

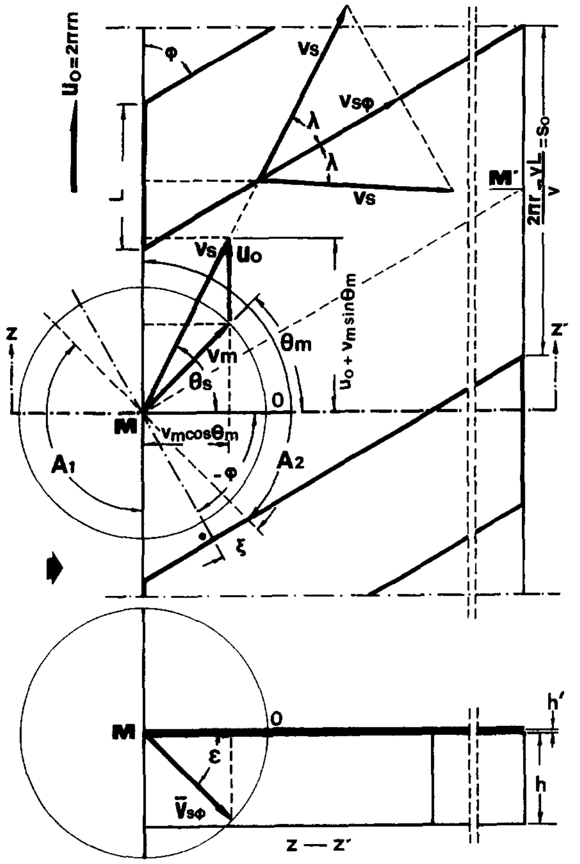

Figure 1. A front part configuration of the helicoid multi-groove vacuum pump.

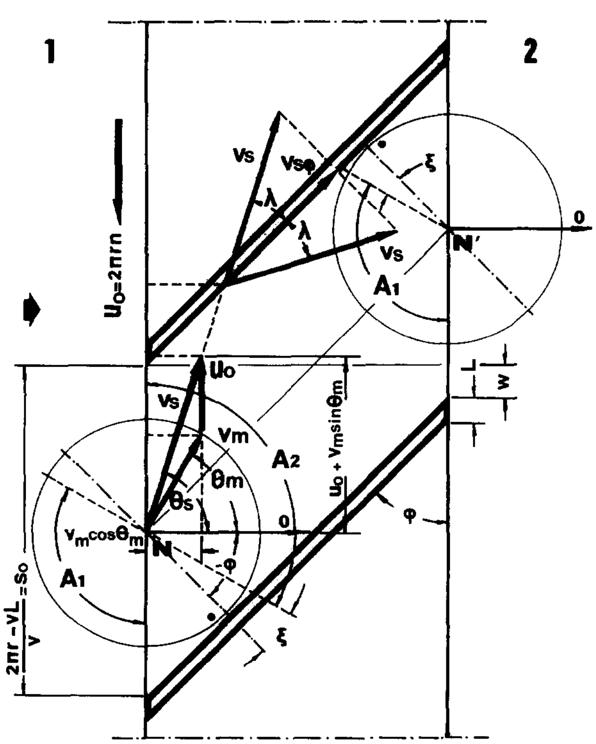

Figure 2. A front part configuration of a disc of the turbomolecular pump.

is normal to the radius of rotation while point N lies on the radius. We assume that the discs having blades are motionless, whereas the point N, and of course the emitted molecules, are moving with velocity $\vec{u}_0$. Thus the final combined velocity of molecules of the gas, which collide on the blades, will be $\vec{V}_S = \vec{V}_m + \vec{u}_0$. Its projection in the pumping direction NN′ is $V_{S\varphi} = V_S \cdot \cos\lambda$.

It is clear now that both cases, in principle, can be treated in a common way.

Since the thermal velocities of molecules of a gas obey Maxwell–Boltzmann distribution law, we can write down the integral[4] which gives the most probable average value of the projections of velocities V_S in the direction MM′ or NN′

$$\bar{V}_{S\varphi} = \frac{1}{S_0\pi a^2}\int_0^{S_0}\int_{V_S=0}^{\infty}\int_{\theta_{S1}}^{\theta_{S2}} V_S\cos\lambda\, e^{-V^2_S/a^2} V_S\, \mathrm{d}V_S \mathrm{d}\theta_S \mathrm{d}s \qquad (1)$$

where $\sqrt{2RT}[m.s^{-1}]$ is the most probable velocity of molecules of the pumping gas and λ is the angle velocity V_S makes with the direction MM′ or NN′. Also, according to Figures 1 and 2, the following relations are true

$$u_0 = 2\pi rn[m\cdot s^{-1}]$$

$$\cos\lambda = \sin(\theta_S+\varphi)$$

$$V_m\cos\theta_m = V_S\cos\theta_S$$

$$V_m\sin\theta_m + u_0 = V_S\sin\theta_S \qquad (2)$$

where:

- r is the average radius of rotation [m]
- n is the number of turns/second of the pumps [s^{-1}]
- θ_s is the angle velocity $\vec{V}_S$ makes with the direction MM′ or NN′
- θ_m is the angle velocity $\vec{V}_M$ makes with the direction MM′ or NN′, and
- φ is the angle of grooves or blades for helicoid or turbomolecular pump, respectively.

Introducing relations (2) into integral (1) we obtain:

$$\bar{V}_{S\varphi} = \frac{1}{S_0\pi a^2}\int_0^{S_0}\int_{V_S=0}^{\infty}\int_{\theta_{S1}}^{\theta_{S2}} V_s\sin(\theta_S+\varphi)e^{-V_S^2/a^2} V_S\,\mathrm{d}V_S\,\mathrm{d}\theta_S\,\mathrm{d}s \qquad (3)$$

$$\bar{V}_{S\varphi} = \frac{1}{S_0\pi a^2}\int_0^{S_0}\int_{V_S=0}^{\infty}\int_{\theta_{S1}}^{\theta_{S2}} \cos\varphi V_S\sin\theta_S e^{-V_S^2/a^2} V_S\,\mathrm{d}V_S\,\mathrm{d}\theta_S\,\mathrm{d}s$$
$$+\frac{1}{S_0\pi a^2}\int_0^{S_0}\int_{V_S=0}^{\infty}\int_{\theta_{S1}}^{\theta_{S2}} \sin\varphi V_S\cos\theta_S e^{-V_S^2/a^2} V_S\,\mathrm{d}V_S\,\mathrm{d}\theta_s\,\mathrm{d}s$$

or

$$\bar{V}_{S\varphi} = \frac{\cos\varphi}{S_0\pi a^2}\int_0^{S_0}\int_{V_m=0}^{\infty}\int_{\theta_{m1}}^{\theta_{m2}} (u_0(u_0+V_m\sin\theta_m)e^{(u_0^2+V_m^2+2u_0V_m\sin\theta_m)/a^2}$$
$$\times V_m\,\mathrm{d}V_m\,\mathrm{d}\theta_m\,\mathrm{d}s + \frac{\sin\varphi}{S_0\pi a^2}\int_0^{S_0}\int_{V_m=0}^{\infty}\int_{\theta_{m1}}^{\theta_{m2}} V^2{}_m\cos\theta_m$$
$$\times e^{u_0^2+V_m^2+2u_0V_m\sin\theta_m)/a^2}\,\mathrm{d}V_m\,\mathrm{d}\theta_m\,\mathrm{d}s \qquad (4)$$

Using the notation:

$$\frac{u_0}{a} = c \quad \text{and} \quad \frac{V_m}{a} = V \tag{5}$$

integral (4) is normalized and through eqn (5) takes the form

$$\bar{V}_{S\varphi} = \frac{a\cos\varphi}{S_0\pi}\int_0^{S_0}\int_{U=0}^{\infty}\int_{\theta_{m1}}^{\theta_{m2}}(c+U\sin\theta_m)$$

$$\times e^{-(C^2+U^2+2CU\sin\theta_m)}U\mathrm{d}U\mathrm{d}\theta_m\mathrm{d}s$$

$$+\frac{a\sin\varphi}{S\pi}\int_0^{S_0}\int_{U=0}^{\infty}\int_{\theta_{m1}}^{\theta_{m2}}U^2{}_m\cos\theta_m$$

$$\times e^{-(C^2+U^2+2CU\sin\theta_m)}\mathrm{d}U\mathrm{d}\theta_m\mathrm{d}s$$

Performing the integration with respect to V and θ_m we finally get:

$$\bar{V}_{S\varphi} = \frac{a}{4S_0}\int_0^{S_0}\left[\cos\varphi\left\{\frac{ce^{-c^2}}{\pi}(\theta_{m2}-\theta_{m1})+\tfrac{1}{2}(\sin 2\theta_{m2}-\sin 2\theta_{m1})\right.\right.$$

$$+\frac{1}{\sqrt{\pi}}\int_{\theta_{m1}}^{\theta_{m2}}\sin\theta_m e^{-c^2\cos^2\theta_m}(1+2c^2\sin^2\theta_m-2c^2)$$

$$\left.\times[1-erf(c\sin\theta_m)\mathrm{d}\theta_m]\right\}+\sin\varphi\left\{\frac{ce^{c^2}}{2\pi}(\cos 2\theta_{m2}-\cos 2\theta_{m1})\right.$$

$$+\frac{1}{\sqrt{\pi}}\int_{\theta_{m1}}^{\theta_{m2}}\cos\theta_m e^{-c^2\cos^2\theta_m}(1+2c^2\sin^2\theta_m)$$

$$\left.\left.\times[1-erf(c\sin\theta_m)]\mathrm{d}\theta_m\right\}\right]\mathrm{d}s \tag{6}$$

To proceed further in the calculation of the integral (6) let us first have a look into the physics of the problem. There are two categories of molecules which participate in the emission through points M or N. The first category refers to molecules which are coming out of pumping space and are entering into the pump, their average probable velocity $\bar{V}_{S\varphi}$ (12) is given by the integral (6) where the limits are $\theta_{m1} = \xi-\varphi$ and $\theta_{m2} = \pi/2$. For the second category of molecules, which are going the reverse way, the corresponding limits of the integral (6) are $\theta_{m1} = \pi-(\varphi-\xi)$ and $\theta_{m2} = 3\pi/2$ giving the most probable average velocity $\bar{V}_{S\varphi}$ (21) of molecules coming out of the pump and entering the pumping space (it is always angle A_1 = angle A_2).

Thus, the final result for the most probable average velocity $\bar{V}_{0S\varphi}$ of molecules of the pumping gas, which is moving in the pumping direction, is given by

$$\bar{V}_{0S\varphi} = \bar{V}_{S\varphi}(12)+\bar{V}_{S\varphi}(21) \tag{7}$$

Equation (7) is true for both cases (helicoid and turbomolecular) and gives the velocity of molecules of the pumping gas at the time when the pumping starts (at the beginning) without considering the motion of molecules caused by pressure difference in the pumping direction.

The angle ξ which appears in the above limits, is determined with the help of Figure 3. According to the first hypothesis (law of cosine) point C is such that the triangles BCF and DCE are similar. Then, according to Thales' theorem we have

$$\frac{AB+BD}{AB} = \frac{BF\tan\xi+DF\tan\xi+AB}{AB} = \frac{S_0}{S}$$

$$\left(\frac{BF}{AB}+\frac{DF}{AB}\right)\tan\xi+1 = \frac{S_0}{S} \tag{8}$$

but $BF = S\sin\varphi$, $DF = S_0\sin\varphi$, $AB = S\cos\varphi$

$$(S_0+S)\tan\varphi\tan\xi = (S_0-S)$$

and finally

$$\xi = \tan^{-1}\left[\frac{(S_0-S)}{(S_0+S)}\cot\varphi\right] \tag{9}$$

where

$$S_0 = \frac{2\pi r - vL}{v}$$

Equation (9) gives the ξ values, i.e. for s values in the range $0 \le s \le s_0$ the values of ξ are such that $(\pi/2)-\varphi \ge \xi \ge 0$.

Let us return back to the expression (6). Dividing by $\cos\varphi$ and using the notation:

$$\bar{u}_{012} = \frac{\bar{V}_{S\varphi}}{\cos\varphi}\text{ when }\theta_{m1} = \xi-\varphi\text{ and }\theta_{m2} = \frac{\pi}{2}, \tag{10}$$

$$\bar{u}_{021} = \frac{\bar{V}_{S\varphi}}{\cos\varphi}\text{ when }\theta_{m1} = \pi-(\varphi-\xi)\text{ and }\theta_{m2} = \frac{3\pi}{2},$$

we obtain the expression

$$\bar{u}_0 = \frac{a}{4S_0}(\bar{u}_{012}+\bar{u}_{021}) \tag{11}$$

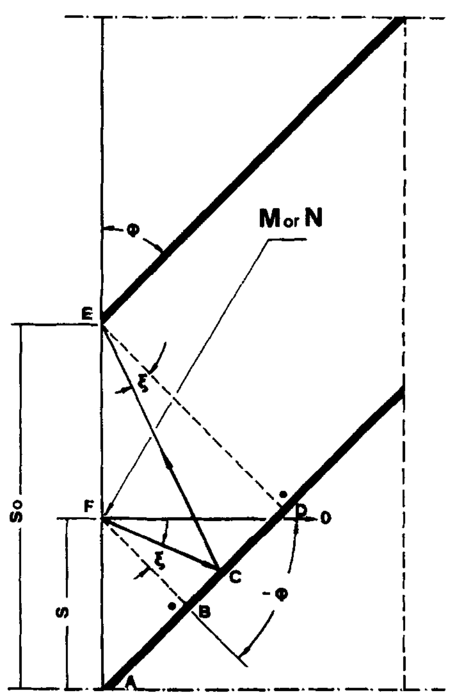

Figure 3. The geometry set related to angle ζ.

which determines the velocity that applies at the place of $u_0 = 2\pi rn$ and reproduces the result given by eqn (7), in agreement with Becker.[2]

As a conclusion, up to now, we can say that the factor $(\bar{u}_{012}+\bar{u}_{021})$ plays the main role in the calculation of the most probable average velocity of molecules in the pumping gas. This factor depends mainly on the angle φ of the grooves or the wings, and the ratio c. To proceed with the integration we substitute $s = \mu s_0$ where $0 \le \mu \le 1$. Then eqn. (9) takes the form

$$\xi = \tan^{-1}\left[\frac{(1-\mu)}{(1+\mu)}\cos\varphi\right] \tag{12}$$

At the same time integral (6) becomes

$$\begin{aligned}\bar{u}'_{\substack{012\\021}} = \frac{a}{4}\int_0^1 \Bigg[\Bigg\{ &\frac{ce^{-c^2}}{\pi}(\theta_{m2}-\theta_{m1})+\frac{1}{2}(\sin 2\theta_{m2}-\sin 2\theta_{m1}) \\ &+\frac{1}{\sqrt{\pi}}\int_{\theta_{m1}}^{\theta_{m2}} \sin\theta_m e^{-C^2\cos^2\theta_m}(1+2c^2\sin^2\theta_m-2c^2) \\ &\times[1-erf(c\sin\theta_m)]\,d\theta_m\Bigg\} + \frac{\sin\theta}{\cos\theta} \\ &\times\Bigg\{\frac{ce^{-c^2}}{\pi}(\cos 2\theta_{m2}-\cos 2\theta_{m1}) \\ &+\frac{1}{\sqrt{\pi}}\int_{\theta_{m1}}^{\theta_{m2}} \cos\theta_m e^{-c^2\cos^2\theta_m}(1+2c^2\sin^2\theta_m) \\ &[1-erf(c\sin\theta_m]d\theta_m\Bigg\}\Bigg]d\mu\end{aligned} \tag{13}$$

and the eqn (11) is modified to

$$\bar{u}_0 = \frac{a}{4}\left(\bar{u}'_{012}+\bar{u}'_{021}\right) \tag{14}$$

By keeping the ratio c in the interval $0 \le c \le 5$ for different values of φ, the coefficient $(\bar{u}'_{012}+\bar{u}'_{021})$ obtain the values presented in Table 1 from which the family of curves of Figure 4 result.

The helicoid multi-groove molecular vacuum pump. It has been shown that, for the helicoid multi-groove molecular vacuum pumps,[5] the compression k and the pumping speed S are given by the expressions:

$$\frac{(2\pi r)^2 h'^2}{\alpha L h^2(2\pi r - vL)v}\cdot\frac{(1+\sigma^2)}{\sigma^2}\frac{(k-1)^2}{k}+\frac{(2\pi r)^2 h'\theta u_0}{2h^2(2\pi r-vL)v}(1+\sigma^2)\cdot\frac{(k^2-1)}{k}-\frac{(2\pi r)\theta u_0}{hv}\ln k+\ln^2 k = 0 \tag{15}$$

and

$$S = (2\pi r - vL)h\frac{\sigma}{(1+\sigma^2)}\left[\frac{u_0}{2}-\frac{hv}{4\pi r\theta}\frac{k-1)}{k}\right] \tag{16}$$

where:

- u_0 is the rotational speed of the pump $[m \times s^{-1}]$,
- r is the mean radius of rotation [m],
- v is the number of grooves,
- L is the front groove distance [m],
- θ is the coefficient of external viscosity $[s \times m^{-1}]$,
- h' is the magnitude of losses between the pump and the moving surface [m],
- h is the height of groove,
- α is a correction coefficient which takes the value of 2, and
- σ $= \tan\varphi$ where φ is the angle of grooves.

Also we show that the average speed of molecules, to the first approximation, is $u_0 = 2\pi rn$.

Now, in a second approximation, after introducing the statistical behavior of molecules (see also Figure 1), the velocity is given by

$$u_0 = \frac{2}{\pi}\int_{-\pi/2}^{\pi/2}\frac{a}{4}\left(\bar{u}'_{012}+\bar{u}'_{021}\right)\cos\varepsilon\, d\varepsilon$$

providing the final form

$$u_0 = \frac{a}{\pi}\left(\bar{u}'_{012}+\bar{u}'_{021}\right) \tag{17}$$

which, multiplied by the factor $\cos\varphi$, reproduces the result of pumping given by eqn (7). Thus, the compression and the pumping speed are given by eqns (15), (16) and (17).

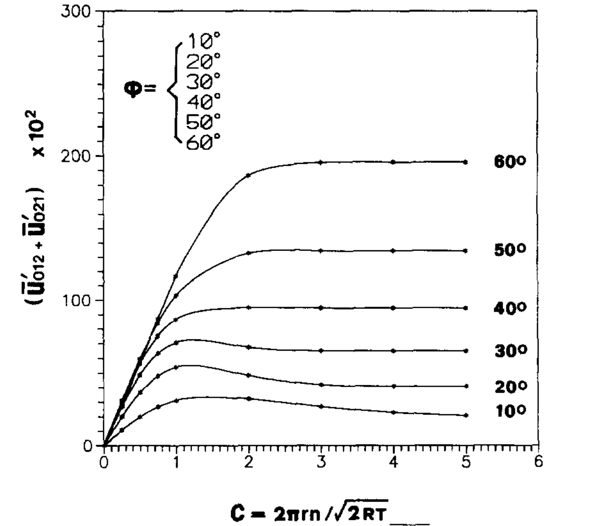

Figure 4. Factor $(u'_{012}+u'_{021})$ vs ratio $c = 2\pi rn/\sqrt{2RT}$ for different angles φ for helicoid or turbomolecular pumps.

Table 1. Values of factor $(\bar{u}'_{012}+\bar{u}'_{021})$ vs ratio $c = 2\pi rn/\sqrt{2RT}$ for different values of angle φ for helicoid or turbomolecular pumps

	φ°					
$c = 2\pi rn/\sqrt{2RT}$	10	20	30	40	50	60
0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.25	0.1064	0.1989	0.2673	0.3052	0.3096	0.2784
0.50	0.1990	0.3663	0.4883	0.5654	0.5968	0.5696
0.75	0.2684	0.4820	0.6359	0.7535	0.8424	0.8720
1.00	0.3121	0.5420	0.7091	0.8676	1.0344	1.1666
2.00	0.3216	0.4851	0.6757	0.9498	0.3281	0.8636
3.00	0.2684	0.4207	0.6520	0.9471	0.3449	0.9528
4.00	0.2296	0.4113	0.6516	0.9471	1.3453	1.9556
5.00	0.2097	0.4108	0.6516	0.9472	1.3456	1.9562

For an optimal practical construction it is true that:

(i) h' is very small, (ii) the term $(hv/4\pi r\theta)(k-1)/k$ contributes less than 20% of the term $(u_0/2)$ in eqns (15) and (16) in the design of pumps it is taken $(2\pi r - vL) = \pi r$. Then eqns (15), (16) and (17) are simplified to:

$$k = \exp\left[\frac{2r\theta a}{hv}\left(\bar{u}'_{012}+\bar{u}'_{021}\right)\right] \tag{18}$$

$$S = \frac{rha}{2}\cdot\frac{\sigma}{(1+\sigma^2)}\left(\bar{u}'_{012}+\bar{u}'_{021}\right) \tag{19}$$

Example 1. Among all geometrical parameters the most important role is played by the angle φ of grooves, the value of which for a specific pump can always ensure the best pair of values for k and S. More specifically, let us consider the set of parameters of a typical, industrial construction[6] with:

$h' = 2.5\times10^{-4}$ m, $n = 1000$ s^{-1}, $\alpha = 2$, $r = 45\times10^{-3}$ m, $L = 23\times10^{-3}$ m, $h = 5\times10^{-3}$ m, $v = 6$, $\theta = 1.61\times10^{-3}$ (s m^{-1}) for air pumping gas, and $a = 412$ (m s^{-1}) at temperature of 20°C.

Then, the system of eqns (15), (16) and (17) becomes

$$5\times10^{-3}\frac{(1+\sigma^2)(k-1)^2}{\sigma^2\quad k}+0.0971\left(\bar{u}'_{012}+\bar{u}'_{021}\right)$$
$$\times(1+\sigma^2)\frac{(k-1)^2}{k}-1.9989\left(\bar{u}'_{012}+\bar{u}'_{021}\right)\ln k+\ln^2 k = 0 \tag{20}$$

$$S = 3.6186\times10^{-4}\frac{\sigma}{1+\sigma^2}\left[131.1437\cdot\left(\bar{u}'_{012}+\bar{u}'_{021}\right)-65.9027\frac{(k-1)}{k}\right] \tag{21}$$

In eqns (20) and (21), varying angle φ in the interval $10°\leq\varphi\leq60°$ and keeping a fixed value of c in the interval $0.25\leq c\leq5$, we obtain the results presented in Table 2 and the corresponding family of curves shown in Figure 5.

From this figure we conclude that the best pair of values for compression and pumping speed corresponds to the value $\varphi \cong 45°$. As a final conclusion we see that this approach increase the value of φ by 10° compared with previous determinations.[5]

The turbomolecular pump. For the turbomolecular pump the compression k and the pumping speed S are given by:[7]

$$\frac{(2\pi r)h'^2 v}{b'(2\pi r-vL)^2}\left(\frac{\sqrt{1+\sigma^2}}{\sigma}\right)^3\frac{(k-1)^2}{k}+\left[\frac{h'}{2}\sigma^2+\frac{b'h''^2v^2)}{(4\pi r)^2}\right]$$
$$\times\frac{(2\pi r)^2\theta u_0}{b'(2\pi r-vL)^2}\left(\frac{\sqrt{1+\sigma^2}}{\sigma}\right)^3\frac{(k^2-1)}{k}$$
$$-\frac{(2\pi r)\theta u_0}{(2\pi r-vL)}\frac{\sqrt{1+\sigma^2}}{\sigma}\ln k+\ln^2 k = 0 \tag{22}$$

$$S = (2\pi r-vL)b'\frac{\sigma}{1+\sigma^2}\left[\frac{u_0}{2}-\frac{(2\pi r-vL}{(8\pi r)\theta}\cdot\frac{\sigma}{\sqrt{1+\sigma^2}}\cdot\frac{(k^2-1)}{k^2}\right] \tag{23}$$

where:

$u_0 = 2\pi rn$ is the rotational speed of the pump [m s^{-1}],
r is the mean radius of the blades [m],
v is the number of blades in each disc,
L is the front blade distance [m],
θ is the coefficient of external viscosity [s m^{-1}],
w is the thickness of the optical opaqueness [m],
h' is the gap loss between rotor and stator [m],
h'' is the distance between the discs [m],
b' is the length of each blade [m], and
σ = tanφ where φ is the angle of the blades.

We took, in a first approximation, the average velocity of molecules to be equal to the rotation velocity times cosφ divided by 2. Thus, introducing the statistical behavior of molecules in our new calculation, the velocity u_0, in eqns (22) and (23), is replaced by that given in eqn (14) times two:

$$u_0 = \frac{a}{2}\left(\bar{u}'_{012}+\bar{u}'_{021}\right) \tag{24}$$

It is obvious that for the best construction, where the losses, originating from the presence of h' and h'', are very small, the eqns (22), (23) and (24) are simplified to:

$$k = \exp\left[\theta u_0\frac{\sqrt{1+\sigma^2}}{\sigma^2}\cdot\frac{1}{\left(1-\frac{vL}{2\pi r}\right)}\right] \tag{25}$$

$$S = b'(2\pi r-vL)\frac{\sigma}{1+\sigma^2}\cdot\frac{u_0}{2}-\frac{b'(2\pi r-vL)^2}{(8\pi r)\theta}\cdot\frac{\sigma^2}{\left(\sqrt{1+\sigma^2}\right)^3}\cdot\frac{(k^2-1)}{k^2} \tag{26}$$

$$u_0 = \frac{a}{2}\left(\bar{u}'_{012}+\bar{u}'_{021}\right) \tag{27}$$

Finally considering the loss

$$\frac{b'(2\pi r-vL)^2}{(8\pi r)}\cdot\frac{\sigma^2}{\left(\sqrt{1+\sigma^2}\right)^3}\cdot\frac{(k^2-1)}{k^2}$$

to be very small, the above result is reduced to the system of equations:

$$k = \exp\left[\frac{\theta a}{2\left(1-\frac{vL}{2\pi r}\right)}\cdot\frac{\sqrt{1+\sigma^2}}{\sigma}\left(\bar{u}'_{012}+\bar{u}'_{021}\right)\right] \tag{28}$$

$$S = b'(2\pi r-vL)\frac{a}{4}\frac{\sigma}{(1+\sigma^2)}\left(\bar{u}'_{012}+\bar{u}'_{021}\right) \tag{29}$$

Example 2. The most important among all parameters is the angle φ, which for a specific choice of a turbomolecular pump ensures the best pair of values for k and S. Let us take the following typical, for an industrial construction,[7] set of parameters:

Table 2. The compression k and the pumping speed S versus φ for several c values in the interval $0.25 \leq c \leq 5$ for a helicoid multi-groove pump

	k values							
	$c = 2\pi rn/\sqrt{2RT}$							
φ°	0.25	0.50	0.75	1.00	2.00	3.00	4.00	5.00
10	1.1776	1.3569	1.5078	1.6106	1.6338	1.5077	1.4215	1.3791
20	1.4012	1.8537	2.2401	2.4670	2.2514	2.0272	1.9962	1.9946
30	1.5715	2.2598	2.8525	3.1899	3.0322	2.9241	2.9223	2.9223
40	1.6439	2.4645	3.2394	3.7858	4.2141	4.1996	4.1996	4.2001
50	1.5881	2.3757	3.2380	4.0154	5.3227	5.4002	5.4021	5.4034
60	1.3949	1.9286	2.5834	3.2707	4.8454	5.0295	5.0351	5.0364

	S values							
	$c = 2\pi rn/\sqrt{2RT}$							
φ°	0.25	0.50	0.75	1.00	2.00	3.00	4.00	5.00
10	2.45×10^{-4}	5.36×10^{-4}	7.97×10^{-4}	9.77×10^{-4}	1.02×10^{-3}	7.97×10^{-4}	6.47×10^{-4}	5.74×10^{-4}
20	8.28×10^{-4}	2.04×10^{-3}	3.08×10^{-3}	3.68×10^{-3}	3.11×10^{-3}	2.51×10^{-3}	2.42×10^{-3}	2.42×10^{-3}
30	1.72×10^{-3}	4.24×10^{-3}	6.31×10^{-3}	7.43×10^{-3}	6.91×10^{-3}	6.55×10^{-3}	6.55×10^{-3}	6.55×10^{-3}
40	2.51×10^{-3}	6.18×10^{-3}	9.42×10^{-3}	1.16×10^{-2}	1.32×10^{-2}	1.31×10^{-2}	1.31×10^{-2}	1.31×10^{-2}
50	2.86×10^{-3}	7.09×10^{-3}	1.15×10^{-2}	1.53×10^{-2}	2.14×10^{-2}	2.17×10^{-2}	2.17×10^{-2}	2.18×10^{-2}
60	2.78×10^{-3}	6.69×10^{-3}	1.15×10^{-2}	1.67×10^{-2}	2.93×10^{-2}	3.17×10^{-2}	3.18×10^{-2}	3.18×10^{-2}

$n = 500\,\mathrm{s}^{-1}$, $r = 75\times10^{-3}\,\mathrm{m}$, $h'' = 0.5\times10^{-3}\,\mathrm{m}$, $h' = 0.25\times10^{-3}\,\mathrm{m}$, $v = 48$, $b' = 30\times10^{-3}\,\mathrm{m}$, $L = 4.3\times10^{-3}\,\mathrm{m}$.

In addition the length of the pump is considered large enough. For air pumping gas at temperature of 20°C, $\theta = 1.61\times10^{-3}\,\mathrm{s\,m^{-1}}$ and $a = 412\,\mathrm{m\,s^{-1}}$. Introducing these parameters in eqns (22), (23) and (24) we get:

$$1.0601\times10^{-3}\left(\frac{\sqrt{1+\sigma^2}}{\sigma}\right)^3\frac{(k-1^2)}{k}$$

$$+(1.25\sigma^2+1.9454\times10^{-1})5.5227\times10^{-3}\left(\bar{u}'_{012}+\bar{u}'_{021}\right)$$

$$\times\left(\frac{\sqrt{1+\sigma^2}}{\sigma}\right)^3\cdot\frac{(k^2-1)}{k}$$

$$-0.7413\cdot\left(\bar{u}'_{012}+\bar{u}'_{021}\right)\frac{\sqrt{1+\sigma^2}}{\sigma}\ln k+\ln^2 k = 0 \tag{30}$$

and

$$S = 3.1626\times10^{-3}\cdot\frac{\sigma}{1+\sigma^2}\left[206\left(\bar{u}'_{012}+\bar{u}'_{021}\right)-138.9485\right.$$

$$\left.\cdot\frac{\sigma}{(\sqrt{1+\sigma^2})}\cdot\frac{(k^2-1)}{k^2}\right] \tag{31}$$

For different c ratios where $0.25\leq c\leq 5$ and for values of φ: $0\leq\varphi\leq 60°$ we obtain the values of Table 3 and the corresponding set of curves of Figure 6. From Figures (6(a)) and (6(b)) we conclude that the best pair of values for compression and pumping speed corresponds to the value $\varphi\cong 35°$, which means that this approach increases the value of φ by 15° compared with previous determinations.[7] The value of $\varphi\cong 35°$ differs from that given by Kruger and Shapiro[3] but it agrees with the one given by R. Frank.[8]

Conclusions

The research area. The introduction of the statistical behavior of molecules, in studying the molecular pumps, requires that the research area be restricted between the pump and the pumping space or at least in the neighborhood of the pump's entrance. In this way, after the dynamic equilibrium is established the probability of a two molecules collision is negligible. However, the general idea of Gaede and Becker to treat the problem is based on the velocity that the molecules have at the entrance of the pump which is equal, in the first approximation, to $u_0 = 2\pi rn\cos\varphi$. Consequently, the introduction into eqns (15), (16), (22) and (23) of the most probable molecular average velocity is permissible. Away from the neighbourhood region and mainly at the end of the pump, where the gas pressure increases, at the same time the probability of a two molecules collision, also increases, and hence the velocity that we calculate in eqn (14) is invalid. The gas is now treated as an homogeneous liquid and eqns (15), (16), (22) and (23) with $u_0 = 2\pi rn\cos\varphi$, are valid.

The factor $(\bar{u}'_{012}+\bar{u}'_{021})$. The factor $(\bar{u}'_{012}+\bar{u}'_{021})$ in eqn (14), referred to the helicoid multi-groove or referred to the turbomolecular cases, takes a zero value for $c = 0$, i.e: $\bar{u}'_{012} = -\bar{u}'_{021} \neq 0$. This means, from a macroscopic point of view, that this factor doesn't express two velocities but it expresses only one velocity which corresponds to a combination of both of them. However, for a turbomolecular pump in dynamic equilibrium we can assume that the pumping gas molecules are developing two different velocities corresponding to the entrance and the exit of each disc. These two velocities are equal to $au'_{012}\cos\varphi/4$ and $au'_{021}\cos\varphi/4$, respectively, in the direction MM′ and NN′ as it is shown in Figures 1 and 2. In this way the first velocity follows

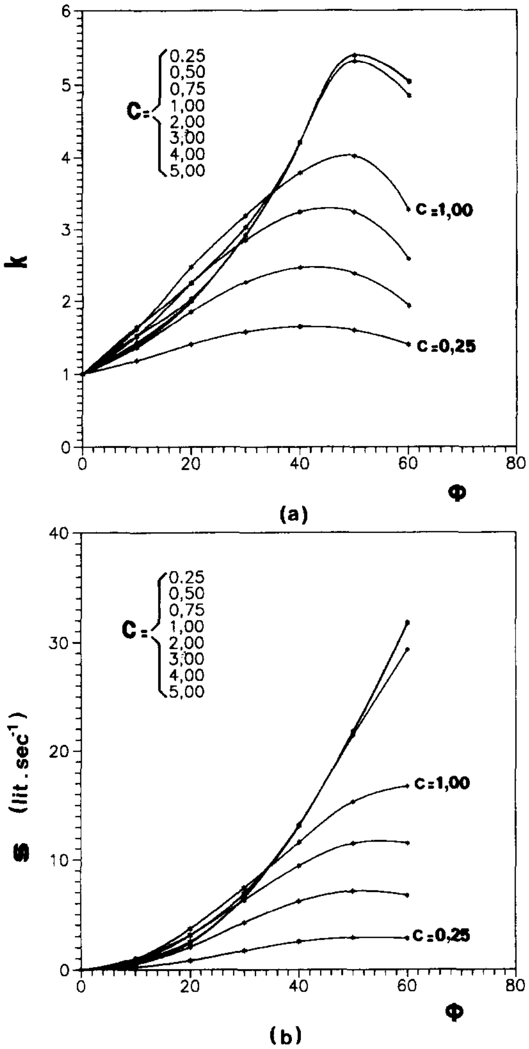

Figure 5. Compression k (a) and pumping speed S (b) vs angle φ for different ratios $c = 2\pi rn/\sqrt{2RT}$ for the helicoid multi-groove vacuum pump.

the direction of pumping whereas the second velocity is opposite to that direction. Under these conditions the amount of gas which goes through the entrance having the same direction as that of the pumping is

$$Q_1 = \frac{a\bar{u}'_{012}}{4\sqrt{1+\sigma^2}}(2\pi r - vL)b'p_1 \tag{32}$$

and the amount of gas which goes through the exit of the disc having opposite direction (according to Figure 2) is

$$Q_2 = \frac{a\bar{u}'_{021}}{4\sqrt{1+\sigma^2}}(2\pi r - vL)b'p_2 \tag{33}$$

where p_1 and p_2 are the pressures in opposite sides of the disc. It is, however, true that $Q_1 = Q_2$, and from eqns (32) and (33) we have:

$$\frac{a\bar{u}'_{012}}{4\sqrt{1+\sigma^2}}(2\pi r - vL)b'p_1 = \frac{a\bar{u}'_{021}}{4\sqrt{1+\sigma^2}}(2\pi r - vL)b'p_2$$

or

$$\frac{p_2}{p_1} = \frac{\bar{u}'_{012}}{\bar{u}'_{021}}$$

The last ratio is precisely the compression k of the pump

$$k = \frac{\bar{u}'_{012}}{\bar{u}'_{021}} \tag{34}$$

Finally, according to eqn (14) the pumping speed of the pump is

$$S = (2\pi r - vL)b'\frac{a\sigma}{4(1+\sigma^2)}\left(\bar{u}'_{012} + \bar{u}'_{021}\right) \tag{35}$$

Equations (34) and (35) are the well known relations of Kruger and Shapiro,[3] in terms of velocity, where in their approximation neither the losses of the system arising from the difference in pressure in opposite sides of the disc nor the peripheral ones are mentioned.

Types of molecular pumps. For the helicoid multi-groove molecular pumps, where the height of grooves changes proportionally with the pump length,[9] eqns (15) and (16) are valid with the replacement:

$$h = \frac{h_2 - h_1}{\ln h_2 - \ln h_1} \tag{36}$$

in accord with Becker, where h_2 is the height of groove in the pump entrance and h_1 is the height of groove in the pump exit.

For the turbomolecular pumps, where the distance changes proportionally with the thickness d'' of the disc[10] (see Figure 7), eqns (22), (23) and (24) must be modified to:

$$u_0 = \frac{a}{4}\left(\bar{u}'_{012} + \bar{u}'_{021}\right) \tag{37}$$

$$\frac{(2\pi r)h'^2}{b'v^2h^2}\left(\frac{\sqrt{1+\sigma^2}}{\sigma}\right)^3\frac{(k-1)^2}{k}$$

$$x + \left[\frac{h'}{2}\sigma^2 + \frac{b'h''^2v^2}{(4\pi r)^2}\right]\frac{(2\pi r)^2\theta u_0}{b'v^2h^2}\left(\frac{\sqrt{1+\sigma^2}}{\sigma}\right)^3\frac{(k^2-1)}{k}$$

$$-\frac{(2\pi r)\theta u_0\sqrt{1+\sigma^2}}{V\hbar\quad\sigma}\ln k + \ln^2 k = 0 \tag{38}$$

$$S = b'vh\frac{\sigma}{1+\sigma^2}\left[\frac{u_0}{2} - \frac{vh}{(8\pi r)\theta}\frac{\sigma}{\sqrt{1+\sigma^2}}\frac{(k^2-1)}{k^2}\right] \tag{39}$$

and the value of h, according to Becker, must be replaced by

$$h = \frac{h_2 - h_1}{\ln h_2 - \ln h_1}$$

with

$$h_2 = \frac{(2\pi r + vL)}{v} + L, \quad h_1 = \frac{(2\pi r + vL)}{v} - L,$$

so that

$$h = \frac{2L}{\ln\frac{2\pi r}{v} - \ln\left(\frac{2\pi r}{v} - 2L\right)} \tag{40}$$

Table 3. The compression k and the pumping speed S versus φ for several c values in the interval $0.25 \leq c \leq 5$. for a turbomolecular pump.

	k values							
	$c = 2\pi rn/\sqrt{2RT}$							
φ°	0.25	0.50	0.75	1.00	2.00	3.00	4.00	5.00
10	1.3949	1.8524	2.2756	2.5805	2.6508	2.2756	2.0300	1.9129
20	1.4921	2.0836	2.6167	2.9410	2.6326	2.3200	2.2774	2.2752
30	1.4600	1.9930	2.4496	2.7117	2.5886	2.5051	2.5037	2.5037
40	1.4005	1.8641	2.2891	2.5908	2.8312	2.8230	2.8230	2.8233
50	1.3282	1.7264	2.1569	2.5630	3.3254	3.3748	3.3760	3.3769
60	1.2451	1.5644	1.9786	2.4792	4.1438	4.4137	4.4224	4.4242
	S values							
	$c = 2\pi rn/\sqrt{2RT}$							
φ°	0.25	0.50	0.75	1.00	2.00	3.00	4.00	5.00
10	5.51×10^{-3}	1.29×10^{-2}	1.94×10^{-2}	2.37×10^{-2}	2.46×10^{-2}	1.89×10^{-2}	1.57×10^{-2}	1.39×10^{-2}
20	1.50×10^{-2}	3.95×10^{-2}	5.97×10^{-2}	7.08×10^{-2}	6.02×10^{-2}	4.88×10^{-2}	4.71×10^{-2}	4.70×10^{-2}
30	2.49×10^{-2}	6.66×10^{-2}	1.00×10^{-1}	1.18×10^{-1}	1.10×10^{-1}	1.04×10^{-1}	1.04×10^{-1}	4.04×10^{-1}
40	2.97×10^{-2}	8.23×10^{-2}	1.29×10^{-1}	1.60×10^{-1}	1.83×10^{-1}	1.82×10^{-1}	1.82×10^{-1}	1.82×10^{-1}
50	2.75×10^{-2}	8.13×10^{-2}	1.40×10^{-1}	1.91×10^{-1}	2.75×10^{-1}	2.80×10^{-1}	2.80×10^{-1}	2.80×10^{-1}
60	2.01×10^{-2}	6.32×10^{-2}	1.23×10^{-1}	1.91×10^{-1}	3.71×10^{-1}	3.95×10^{-1}	3.95×10^{-1}	3.95×10^{-1}

Table 4. The compression k and the pumping speed S versus φ for several c values in the interval $0.25 \leq c \leq 5$. for a turbomolecular pump of Figure 7. with data taken from paragraph 2.3 and eqn (40)

	k values							
	$c = 2\pi rn/\sqrt{2RT}$							
φ°	0.25	0.50	0.75	1.00	2.00	3.00	4.00	5.00
10	1.4101	1.8879	2.3307	2.6494	2.7228	2.3307	2.0725	1.9513
20	1.5275	2.1737	2.7634	3.1245	2.7811	2.4346	2.3875	2.3850
30	1.4946	2.0752	2.5869	2.8805	2.7428	2.6489	2.6473	2.6473
40	1.4303	1.9376	2.4088	2.7461	3.0163	3.0070	3.0070	3.0074
50	1.3517	1.7853	2.2602	2.7124	3.5692	3.6250	3.6264	3.6274
60	1.2615	1.6064	2.0591	2.6114	4.4669	4.7687	4.7784	4.7805
	S values							
	$c = 2\pi rn/\sqrt{2RT}$							
φ°	0.25	0.50	0.75	1.00	2.00	3.00	4.00	5.00
10	5.41×10^{-3}	1.25×10^{-2}	1.87×10^{-2}	2.28×10^{-2}	2.37×10^{-2}	1.87×10^{-2}	1.52×10^{-2}	1.34×10^{-2}
20	1.48×10^{-2}	3.84×10^{-2}	5.77×10^{-2}	6.83×10^{-2}	5.83×10^{-2}	4.73×10^{-2}	4.57×10^{-2}	4.57×10^{-2}
30	2.45×10^{-2}	6.49×10^{-2}	9.71×10^{-2}	1.14×10^{-1}	1.06×10^{-1}	1.01×10^{-1}	1.01×10^{-1}	1.01×10^{-1}
40	2.93×10^{-2}	8.04×10^{-2}	1.25×10^{-1}	1.55×10^{-1}	1.77×10^{-1}	1.76×10^{-1}	1.76×10^{-1}	1.76×10^{-1}
50	2.72×10^{-2}	7.95×10^{-2}	1.36×10^{-1}	1.85×10^{-1}	2.65×10^{-1}	2.70×10^{-1}	2.70×10^{-1}	2.70×10^{-1}
60	1.98×10^{-2}	6.20×10^{-2}	1.20×10^{-1}	1.85×10^{-1}	3.56×10^{-1}	3.78×10^{-1}	3.79×10^{-1}	3.79×10^{-1}

Example 3. Using in eqn. (40) the numerical data of the section entitled 'The turbomolecular pump' is obtained $h = 4.12\times10^{-3}$ m. Then, eqns (38) and (39) take the form

$$1.2049\times10^{-3}\left(\frac{\sqrt{1+\sigma^2}}{\sigma}\right)^3\frac{(k-1)^2}{k}$$

$$+[1.25\sigma^2+1,9454\times10^{-1}]6.2774\times10^{-3}\left(\bar{u}'_{012}+\bar{u}'_{021}\right)$$

$$\times\left(\frac{\sqrt{1+\sigma^2}}{\sigma}\right)^3\left(\frac{k^2-1}{k}\right)-0.7903\left(\bar{u}'_{012}+\bar{u}'_{021}\right)$$

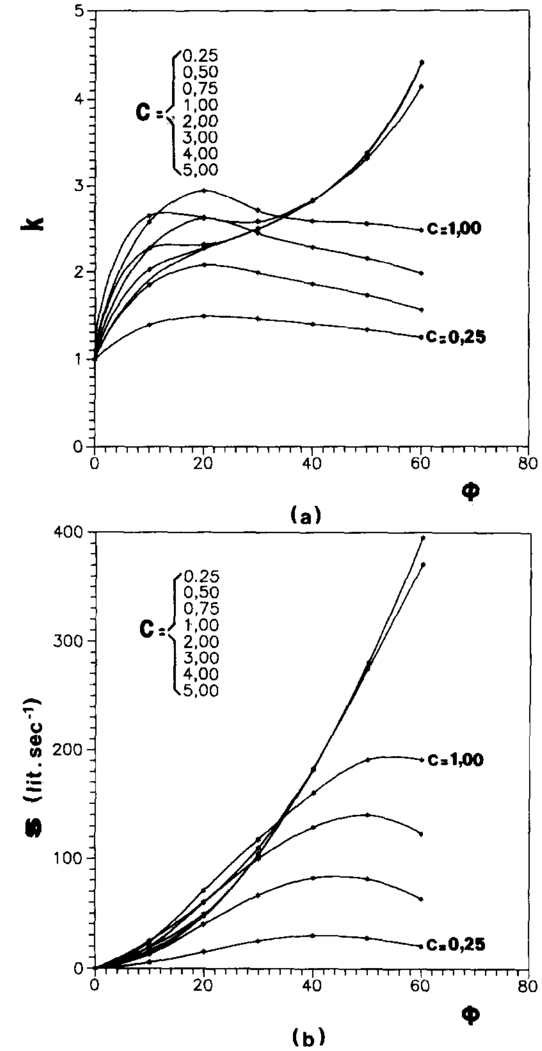

Figure 6. Compression k (a) and pumping speed S (b) vs angle φ for different values of $c = 2\pi rn/\sqrt{2RT}$ for the turbomolecular pump.

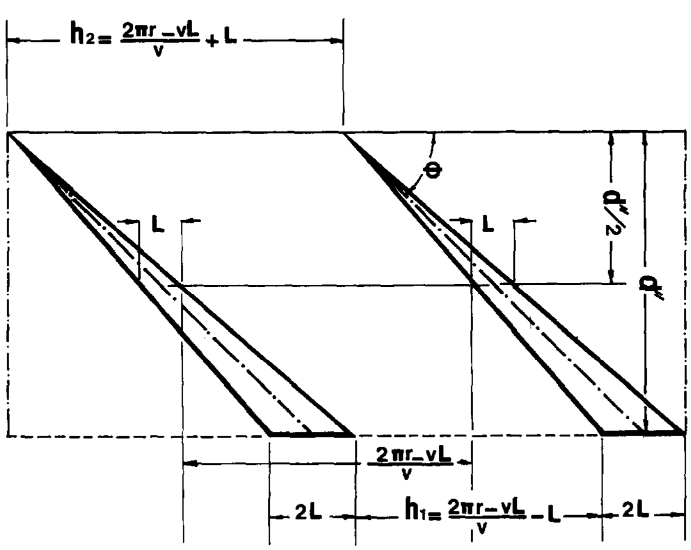

Figure 7. A front part disc configuration for a turbomolecular vacuum pump showing the case of proportionality between the front blade distance L and thickness d''.

$$\times\frac{\sqrt{1+\sigma^2}}{\sigma}\ln k+\ln^2 k = 0 \tag{41}$$

$$S = 2.9664\times 10^{-3}\frac{\sigma}{1+\sigma^2}\left[206\left(\bar{u}'_{012}+\bar{u}'_{021}\right)\right.$$

$$\left.-130.3291\frac{\sigma}{\sqrt{1+\sigma^2}}\frac{(k^2-1)}{k^2}\right] \tag{42}$$

Table 4 and the set of curves of Figure 8 show the compression k and the pumping speed S vs φ: $10 \leq \varphi \leq 60°$, for several c values in the interval $0.25 \leq \varphi \leq 5$, for a turbomolecular pump of Figure 7, with data taken from the section entitled 'The turbomolecular pump' and eqn (40).

In our specific example, identical parameters have been used for both types of turbomolecular pumps shown in Figures 2 and 7, and the average distance between blades have been taken $(2\pi r - vL)/v$. As a result we see that the angle φ which gives the best pair of values for compression and the pumping speed in both types is the same of that 35°. The compression is larger for

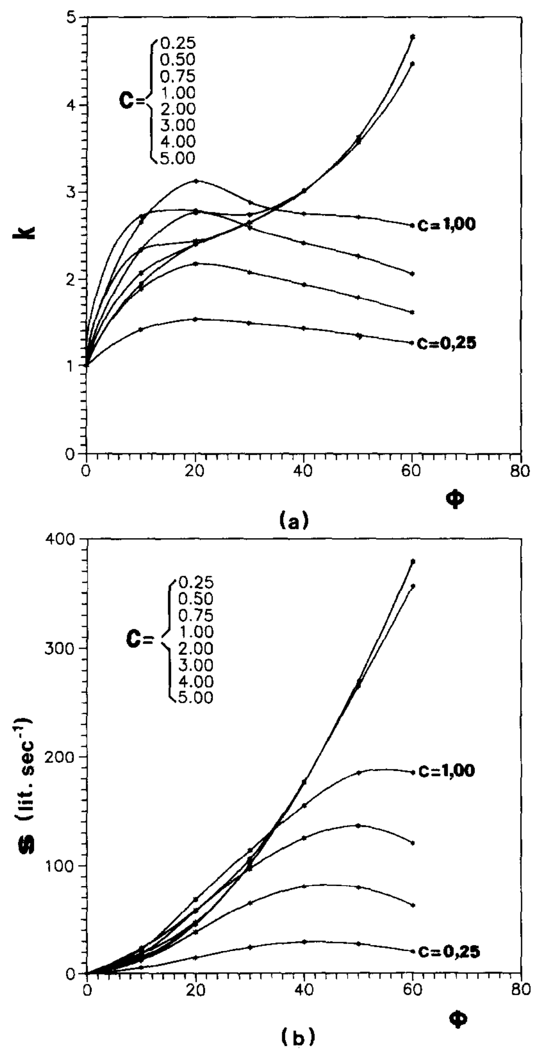

Figure 8. Compression k (a) and pumping speed S (b) vs angle φ for several C values in the interval $0.25 \leqslant C \leqslant 5$ for a turbomolecular pump of Figure 7, with data taken from Section entitled 'The turbomolecular pump' and eqn (40).

pump of Figure 7 whereas the corresponding pumping speed is smaller.

Finally we must point out that the introduction of statistics in the theory of turbomolecular pumps affects calculations of the speed of molecules and the acceptable level of losses, produces results for the compression k and the pumping speed S smaller by a factor of 2. This is in good agreement with the results of Kruger and Shapiro and Frank. Also we must point out that now there exists a way of studying the differences between the helicoid pumps and the turbomolecular pumps. At the same time we must emphasize that the helicoid multi-groove vacuum pumps present the advantage of obtaining larger compression than that of turbomolecular pumps.

In conclusion, we believe, that using the statistical behavior of molecules, identification of the approaches of Gaede and Becker and Kruger and Shapiro has been achieved finally.

References

1. W Gaede, Die Molekularloftpumpe, *Ann Physik*, **41,** 342 (1913).
2. W Becker, Die Turbomolekularpumpe. *Vakuum-Technik*, Jahrang 15, **211** (1966); W Becker, The turbomolecular pump, its design, operation and theory, *Vacuum*, **16** (1966) 625.
3. G Kruger and A Shapiro, Rarefied gas dynamics, in *Proc 2nd Int Symp Calif* (1960), L Talbot (Ed.), Academic Press (1961) 117; G Kruger and A Shapiro, Vacuum pumping with a bladed axial-flow turbo-machine, *7th Nat Symp Vac Techn Transact*, (1960) 6-12.
4. E Sears and G Salinger, *Thermodynamics, Kinetic Theory, and Statistical Thermodynamics*, 3rd Edn, Addison-Wesley, USA (1975).
5. C N Panos, A G Antoniou and S E Valamontes, The helicoid multi-groove vacuum pump in both viscous and molecular states, *Vacuum*, **45,** 841 (1994).
6. J Henning, Thirty years of turbomolecular pumps: A review and recent developments, *J Vac Sci Technol*, **A6(3),** 1196 (1988).
7. A G Antoniou, S E Valamontes and C N Panos, The turbomolecular pump in molecular state, *Vacuum*, **46,** 709 (1995).
8. R Frank, *Konstruktiver Aufbau und mathematische Auslegung von Turbomolekularpu-mpen des Typs TURBOVAC mit senkrecht angeordnetem Axialkompressor. Vakuum Technik*, Jahrgang 23, *Heft*, **4,** 109 (1974).
9. M Holweck, Pompe moleculaire helicoidale, *CR Acad Sci Paris*, **177,** 43 (1923).
10. S Iida and O Kimura, On performance improvement of axial-flow molecular pump, in *Proc 6th Inter Vacuum Congress*, 8 (1974).