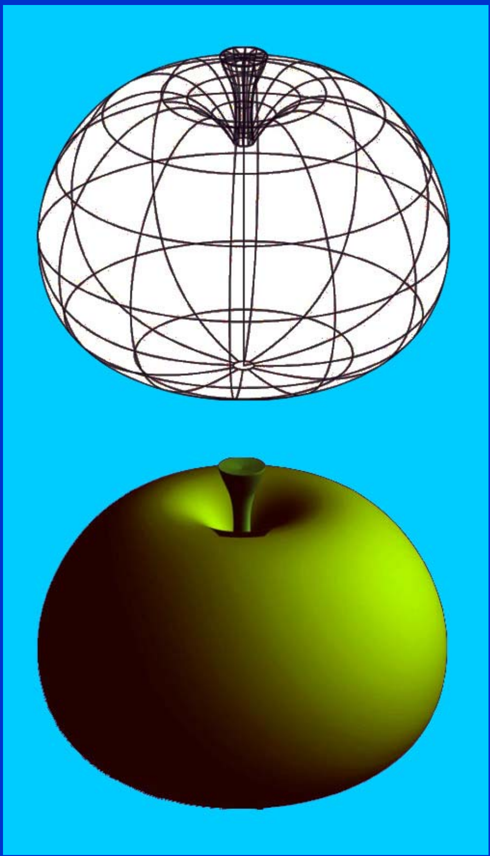

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# Newton's Bulletin

## External Quality Evaluation Reports of EGM08

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# External Quality Evaluation Reports of EGM08

## Special Issue: Newton's Bulletin N. 4

# Foreword

The Joint Working Group (JWG) between the International Gravity Field Service (IGFS) and the Commission 2 of the International Association of Geodesy (IAG), entitled "**Evaluation of Global Earth Gravity Models**", was officially established in 2005. The main objective of this JWG is to study standard validation/calibration techniques for global geopotential models, and to perform quality assessment procedures of GRACE, CHAMP and GOCE based satellite-only and combined solutions for the static part of Earth's gravity field. The external data sets that are commonly used for such purposes include GPS and leveling height data, airborne and surface gravity data, mean oceanographic sea-surface-topography (SST) models and altimetric data, orbit data from other geodetic and altimetric satellites, and astro-geodetic vertical deflections. The initial membership of the JWG included 24 scientists from 15 countries, which has finally increased to 30 scientists from 20 countries due to the strong international interest in evaluating the PGM2007A model, a preliminary version of the official Earth Gravitational Model 2008 (EGM2008).

The IGFS/IAG JWG has successfully coordinated the evaluation of both PGM2007 and EGM2008, in close collaboration with the EGM development team from the U.S. National Geospatial-Intelligence Agency (NGA). This joint evaluation project was carried out through three phases: the implementation and testing of the NGA software for spherical harmonic synthesis using ultra-high degree geopotential models (2006-2007), the evaluation of the PGM2007 model (2007-2008), and finally the evaluation of the official EGM2008 model (2008-2009). Most of the results of the above tasks are publicly available at the official webpage of the working group: http://users.auth.gr/~kotsaki/IAG_JWG/IAG_JWG.html.

The first splinter meeting of the JWG was held on July 31, 2006 in Istanbul during the first IGFS international symposium, and it marked the end of Phase 1. The PGM2007A model was released to the members of the JWG in July 2007, initiating the beginning of Phase 2. A total of thirty evaluation reports for PGM2007A were completed and published at the JWG's website by December 2007. Phase 3 started right after the official release of EGM2008 at the EGU General Assembly in April 2008. The first results of the EGM2008 evaluation tests were presented by the working group members in a dedicated session during the IAG international symposium 'Geoid, Gravity and Earth Observation' that was held in Chania, Greece, June 23-27, 2008.

This special issue of *Newton's Bulletin* consists of 25 peer-reviewed evaluation papers of EGM2008 (and partially of PGM2007A), which are grouped into four different sections according to the geographical region of the evaluation tests: Global, the Americas, Europe and Africa, and Asia, Australia and Antarctica. Their results provide a thorough external assessment of EGM2008, using a variety of geodetic data and testing methodologies.

We are grateful to all people who made the publication of this special issue possible. First of all, we would like to express our deep appreciation to all contributing authors of the evaluation papers for their interest and dedication to the project. The success of this project is primarily attributed to their continuous participation and close cooperation. Secondly, we would like to thank the development team of EGM2008 for their support and continuous collaboration towards the successful completion of this international project. Last but not least, the IGFS and the Commission 2 of the IAG are acknowledged for their effective international leadership, guidance and coordination.

Special thanks are due to the International Geoid Service (IGeS) and the Bureau Gravimétrique International (BGI) for the publication of this special issue of *Newton's Bulletin*.

Jianliang Huang,
*Geodetic Survey Division, CCRS, NRCAN, Canada*

Christopher Kotsakis
*Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Greece*

# Evaluation of EGM08 based on GPS and orthometric heights over the Hellenic mainland (*)

*C. Kotsakis and K. Katsambalos*

Department of Geodesy and Surveying
School of Engineering
Aristotle University of Thessaloniki
Univ. Box 440, Thessaloniki 541 24, Greece

*M. Gianniou*

Ktimatologio S.A. (Hellenic Cadastre)
339 Mesogion Ave.
Athens 152 31, Greece

## ABSTRACT

This report presents the evaluation results for the new Earth Gravitational Model (EGM08) that was recently released by the US National Geospatial-Intelligence Agency, using GPS and leveled orthometric heights in the area of Greece. Detailed comparisons of geoid undulations obtained from the EGM08 model and other combined global geopotential models (GGMs) with GPS/leveling data have been performed in both absolute and relative sense. The test network covers the entire part of the Hellenic mainland and it consists of more than 1500 benchmarks which belong to the Hellenic national triangulation network, with direct leveling ties to the Hellenic vertical reference frame. The spatial positions of these benchmarks have been recently determined at cm-level accuracy (with respect to ITRF2000) during a nation-wide GPS campaign that was organized in the frame of the HEPOS project. Our results reveal that EGM08 offers a major improvement (more than 60%) for the agreement among geoidal, ellipsoidal and orthometric heights over the mainland part of Greece, compared to the performance of other combined GGMs for the same area.

(*) Final report submitted to the IAG/IGFS Joint Working Group "*Evaluation of Global Earth Gravity Models*" (November 2008)

## 1. INTRODUCTION

The development of the Earth Gravitational Model EGM08 by the US National Geospatial-Intelligence Agency (Pavlis et al. 2008) unveiled a major achievement in global gravity field mapping. For the first time in modern geodetic history, a spherical harmonic model complete to degree and order 2159, with additional spherical harmonic coefficients (SHCs) extending up to degree 2190 and order 2159, is available for the representation of the Earth's external gravitational potential. This new model offers an unprecedented level of spatial sampling resolution (~ 9 km) for the recovery of gravity field functionals over the entire globe, and it contributes in a most successful way to the continuing efforts of the geodetic community for a high-resolution and high-accuracy reference model of Earth's mean gravity field.

Following the official release of EGM08 to the Earth science community, there is a strong interest among geodesists to quantify its actual accuracy with different validation techniques and 'external' data sets, independently of the estimation and error calibration procedures that were used for its development. In response to the above interest and as part of the related activities that have been coordinated by the IAG/IGFS Joint Working Group on the *Evaluation of Global Earth Gravity Models*, the objective of this report is to present the EGM08 evaluation results that have been obtained for the area of Greece using GPS and leveled orthometric heights. A brief summary of these results has already been given in Kotsakis et al. (2008), lacking though a number of additional tests that are presented for the first time herein (see Sect. 4).

The test network consists of 1542 control points that belong to the Hellenic national triangulation frame, with direct ties to the Hellenic national vertical reference frame through spirit (and in some cases trigonometric) leveling surveys. These control points were recently re-surveyed through a national GPS campaign in the frame of the HEPOS project (more details to be given in Sect. 2) and their spatial positions have been estimated anew at cm-level accuracy with respect to ITRF2000.

Some key features of our study are the extensive national coverage and high spatial density of the test network, corresponding approximately to an average distance of 7 km between adjacent points throughout Greece (Figure 1). These characteristics have been most helpful in identifying the significant improvement that EGM08 yields, over other existing geopotential models, for the representation of gravity field features in certain Hellenic mountainous areas (see Sect. 3). This is actually the first time that a detailed quality analysis for the performance of global geopotential models (GGMs) is carried out over the entire Hellenic mainland with the aid of precise GPS positioning. Consequently, our study also provides a preliminary, yet reliable, assessment about the feasibility of EGM08 for determining orthometric height differences via GPS/geoid-based leveling techniques in Greece (see Sect. 5).

## 2. DATA SETS

All the evaluation tests and their corresponding results that are presented in the following sections refer to a network of 1542 GPS/leveling benchmarks which covers the entire mainland region of Greece with a relatively uniform spatial distribution (see Figure 1).

Note that some control points which were originally existing in this network, but they were later identified as 'problematic' (mainly due to suspected blunders in their orthometric heights that are provided by the Hellenic Military Geographic Service), have been removed from the following analysis and they are not included in the test network shown in Figure 1.

Although a large number of additional GPS/leveling benchmarks were also available in the Greek islands, they have been deliberately excluded from our current analysis to avoid misleading systematic effects in the evaluation results due to unknown vertical datum differences that exist between the various islands and the mainland region.

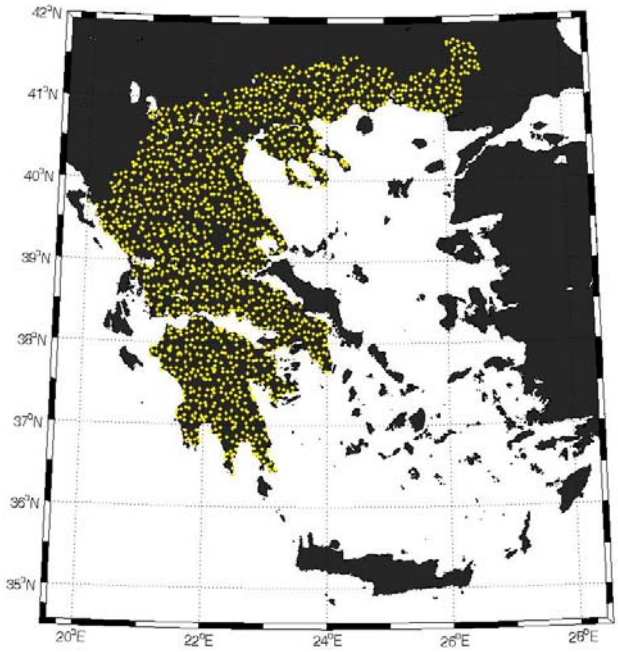


**Figure 1.** Geographical distribution of the 1542 GPS/leveling benchmarks over the Hellenic mainland.

*2.1 Ellipsoidal heights*

Within the frame of currently ongoing efforts for the enhancement of the spatial data infrastructure in Greece, a national GPS campaign took place in 2007 in order to acquire a sufficient number of control points with accurately known 3D spatial positions in an ITRF-type coordinate system. These activities have been initiated by the Ministry for the Environment, Planning and Public Works and the financial support of the EU and the Hellenic State, and they are part of the HEPOS (Hellenic Positioning System) project that will lead to the launch of a modern satellite-based positioning service for cadastral, mapping, surveying and other geodetic applications in Greece (Gianniou 2008). The entire project is coordinated by Ktimatologio S.A, a state-owned private sector firm that is responsible for the operation of the Hellenic Cadastral system.

The aforementioned GPS campaign involved more than 2450 geodetic benchmarks within the existing national triangulation network, part of which are the 1542 points shown in Figure 1. The main scope of the campaign was to provide an ample number of control stations for determining a precise datum transformation model between the official Hellenic Geodetic Reference Frame of 1987 and other ITRF/ETRF-type frames. The actual fieldwork was performed within a 6-month period (March to September 2007) using twelve dual-frequency Trimble 5700/5800 GPS receivers with Zephyr or R8 internal antennas. Thirty three points were used as 'base' reference stations with 24-hour continuous GPS observations, while the rest of the control points were treated as 'rover' stations with observation periods ranging between 1-3 hours. In all cases, a 15-sec sampling rate and an 15° elevation cut-off angle were used for the data collection. Note that the maximum GPS-baseline length that was obtained from the above procedure did not exceed 35 km.

After the processing of the GPS observations using EUREF/EPN ties and IGS precise orbits, the geocentric Cartesian coordinates of all stations (including the 1542 points shown in Figure 1) were determined in ITRF2000 (epoch: 2007.236) and their geometric heights were subsequently derived with respect to the GRS80 ellipsoid. The accuracy of the ellipsoidal heights ranges between 2-5 cm, while the horizontal positioning accuracy with respect to ITRF2000/GRS80 is marginally better by 1-2 cm ($1\sigma$ level).

### *2.2 Orthometric heights*

Helmert-type orthometric heights at the 1542 test points have been determined through leveling ties to surrounding benchmarks of the national vertical reference frame. These local survey ties were performed in previous years by the Hellenic Military Geographic Service (HMGS) using spirit and/or trigonometric leveling techniques. It should be mentioned that a large number of the test points is located in highly mountainous areas (i.e. 24% of them have orthometric heights $H > 800$ m).

The quality of the known orthometric heights in our test network is mainly affected by two factors: the internal accuracy and consistency of the Hellenic vertical datum (HVD), and the observation accuracy of the local leveling ties to the surrounding HVD benchmarks. Due to the absence of sufficient public documentation from the part of HMGS, the absolute accuracy of these orthometric heights is largely unknown. Their values refer, in principle, to the equipotential surface of Earth's gravity field that coincides with the mean sea level at the HVD's fundamental tide-gauge reference station located in Piraeus port (unknown $W_o$ value, period of tide gauge measurements: 1933-1978); for more details, see Antonopoulos et al. (2001), Takos (1989).

### *2.3 GPS-based geoid undulations*

Based on the known ellipsoidal and orthometric heights, geoid undulations have been computed at the 1542 test points according to the equation

$$N^{GPS} = h - H \tag{1}$$

The above values provide the 'external' dataset upon which the following EGM08 validation tests will be performed.

Note that low-pass filtering or other smoothing techniques have <u>not</u> been applied to the GPS/H geoid heights ($N^{GPS}$). As a result, the effect of the *omission error* associated with all tested GGMs will be directly reflected in our evaluation results.

## 2.4 GGM-based geoid undulations

Geoid undulations have also been computed at the 1542 GPS/leveling benchmarks using several different GGMs. For the evaluation results presented herein, we consider the most recent 'mixed' GGMs that have been produced from the combined analysis of various types of satellite data (CHAMP, GRACE, SLR), terrestrial gravity data, and altimetry data; see Table 1.

**Table 1.** GGMs used for the tests at the 1542 Hellenic GPS/leveling benchmarks.

| *Models* | $n_{max}$ | *Reference* |
|---|---|---|
| EGM08 | 2190 | Pavlis et al. (2008) |
| EIGEN-GL04C | 360 | Förste et al. (2006) |
| EIGEN-CG03C | 360 | Förste et al. (2005) |
| EIGEN-CG01C | 360 | Reigber et al. (2006) |
| GGM02C | 200 | Tapley et al. (2005) |
| EGM96 | 360 | Lemoine et al. (1998) |

The determination of GGM geoid undulations was carried out through the general formula (Rapp 1997)

$$N = \zeta + \frac{\Delta g^{FA} - 0.1119H}{\bar{\gamma}} H + N_o \tag{2}$$

where $\zeta$ and $\Delta g^{FA}$ denote the height anomaly and free-air gravity anomaly signals, which are computed from spherical harmonic series expansions (up to $n_{max}$) based on the SHCs of each model and the GRS80 normal gravity field parameters. Only the gravitational potential coefficients with degrees $n \geq 2$ were considered for these harmonic synthesis computations, excluding the contribution of the zero/first-degree harmonics from the GGM-based signals. Note that EIGEN-CG01C and EIGEN-CG03C are the only models among the tested GGMs which are accompanied by non-zero first-degree SHCs. Nevertheless, their omission in the computation of the $\zeta$ values has a negligible effect (mm-level) in our evaluation tests.

The term $N_o$ represents the contribution of the zero-degree harmonic to the GGM geoid undulations with respect to the GRS80 reference ellipsoid. It is computed according to the well known formula (e.g. Heiskanen and Moritz 1967)

$$N_o = \frac{GM - GM_o}{R\gamma} - \frac{W_o - U_o}{\gamma} \tag{3}$$

where the parameters $GM_o$ and $U_o$ correspond to the Somigliana-Pizzeti normal gravity field generated by the GRS80 ellipsoid (Moritz 1992)

$GM_o = 398600.5000 \times 10^9 \text{ m}^3 \text{ s}^{-2}$

$U_o = 62636860.85 \text{ m}^2 \text{ s}^{-2}$

The Earth's geocentric gravitational constant ($GM$) and the constant gravity potential of the geoid ($W_o$) have been set to the following values

$GM = 398600.4415 \times 10^9$ m$^3$ s$^{-2}$

$W_o = 62636856.00$ m$^2$ s$^{-2}$ *(IERS Conventions 2003)*

while the mean Earth radius $R$ and the mean normal gravity $\gamma$ on the reference ellipsoid are taken equal to 6371008.771 m and 9.798 m s$^{-2}$, respectively (GRS80 values). Based on the above conventional choices, the zero-degree term from Eq. (3) yields the value $N_o$ = -0.442 m, which has been added to the geoid undulations obtained from the corresponding SHC series expansions of all GGMs.

***Remark.*** *The numerical computations for the spherical harmonic synthesis of the N values from the various GGMs have been performed with the 'harmonic_synth_v02' software program that is freely provided by the EGM08 development team (Holmes and Pavlis 2006). Note also that the final GGM geoid undulations obtained from Eq. (2) refer to the zero-tide system, with respect to a geometrically fixed reference ellipsoid (GRS80).*

### *2.5 Height data statistics*

The statistics of the individual height datasets that will be used in our evaluation tests are given in Table 2. Note that the statistics for the GGM geoid undulations refer to the values computed from Eq. (2) at the 1542 GPS/leveling benchmarks using the *full* spectral resolution of each model.

From the following table (see, in particular, the *mean* values in the fourth column) it is evident the existence of a large discrepancy (> 25 cm) between the reference surface of the Hellenic vertical datum (which is associated with an unknown $W_o$ value) and the equipotential surface of Earth's gravity field that is specified by the IERS conventional value $W_o$ = 62636856.00 m$^2$ s$^{-2}$ and realized by the various GGMs over the Hellenic mainland region.

**Table 2.** Statistics of various height datasets over the test network of 1542 Hellenic GPS/leveling benchmarks (units in m).

| | *Max* | *Min* | *Mean* | $\sigma$ |
|---|---|---|---|---|
| $h$ | 2562.753 | 24.950 | 545.676 | 442.418 |
| $H$ | 2518.889 | 0.088 | 510.084 | 442.077 |
| $N^{GPS} = h-H$ | 43.864 | 19.481 | 35.592 | 5.758 |
| $N$ (EGM08) | 44.374 | 19.663 | 35.968 | 5.800 |
| $N$ (EIGEN-GL04C) | 44.104 | 19.303 | 35.874 | 5.878 |
| $N$ (EIGEN-CG03C) | 44.049 | 19.257 | 35.861 | 5.867 |
| $N$ (EIGEN-CG01C) | 44.108 | 19.663 | 35.823 | 5.873 |
| $N$ (GGM02C) | 44.034 | 19.771 | 35.905 | 5.780 |
| $N$ (EGM96) | 44.007 | 19.687 | 36.037 | 5.753 |

It is also interesting to observe the considerable mean offset of the full-resolution EGM08 geoid ($n_{max}$ = 2190) with respect to the geoid realizations obtained from other GGMs at the GPS/leveling benchmarks. This offset varies from 6 to 15 cm and it should be attributed to long/medium-wavelength systematic differences between EGM08 and the other GGMs over the Hellenic area.

## 3. POINTWISE EVALUATION TESTS AFTER A SIMPLE BIAS FIT

A series of GGM evaluation tests were performed based on the point values for the ellipsoidal and orthometric heights in the control network. The statistics of the differences between the GPS-based and the GGM-based geoid heights are given in Table 3. In all cases, the values shown in this table refer to the statistics after a least-squares *constant bias fit* was applied to the original misclosures *h-H-N* at the 1542 Hellenic GPS/leveling benchmarks.

The differences in the estimated bias obtained from each model (see last column in Table 3) indicate the existence of systematic regional offsets among the GGM geoids that are likely caused by long/medium-wavelength commission errors in their SHCs and additional omission errors due to their limited spectral resolution. Furthermore, the actual magnitude of the bias between $N^{GPS}$ and $N$ suggests the presence of a sizeable offset between **(a)** the equipotential surface associated with the IERS conventional value $W_o$ = 62636856.00 $m^2s^{-2}$ and realized by the various GGMs over the Hellenic region, and **(b)** the HVD reference surface that is realized through the GPS/H geoid heights $N^{GPS}$ at the test points. For example, based on the results from the full-resolution version of the new model, the HVD reference surface appears to be located 38 cm below the EGM08/$W_o$/GRS80 geoid realization.

**Table 3.** Statistics of the residuals $N^{GPS}$–*N (after a least-squares constant bias fit)* at the 1542 GPS/leveling benchmarks (units in m).

| | *Max* | *Min* | $\sigma$ | *Bias* |
|---|---|---|---|---|
| EGM08 *($n_{max}$=2190)* | 0.542 | -0.437 | 0.142 | -0.377 |
| EGM08 *($n_{max}$=360)* | 1.476 | -1.287 | 0.370 | -0.334 |
| EIGEN-GL04C *($n_{max}$=360)* | 1.773 | -1.174 | 0.453 | -0.283 |
| EIGEN-CG03C *($n_{max}$=360)* | 1.484 | -1.173 | 0.453 | -0.270 |
| EIGEN-CG01C *($n_{max}$=360)* | 1.571 | -1.135 | 0.492 | -0.231 |
| GGM02C *($n_{max}$=200)* | 2.112 | -1.472 | 0.551 | -0.313 |
| EGM96 *($n_{max}$=360)* | 1.577 | -1.063 | 0.423 | -0.446 |

From the results given in the above table, it is evident that EGM08 offers a remarkable improvement for the agreement among ellipsoidal, orthometric and geoidal heights in Greece. Compared to other GGMs, the standard deviation of the EGM08 residuals $N^{GPS}$-*N* over the test network decreases by a factor of 3 (or more). The improvement obtained from the new model is visible even in its 30′ limited-resolution version ($n_{max}$=360), which matches the GPS/H geoid within ±37 cm (in an average pointwise sense), while all previous GGMs of similar resolution do not perform better than ±42 cm. The major contribution, however, comes from the ultra-high frequency band of EGM08 (360 < $n$ < 2190) which enhances the consistency between GGM and GPS/H geoid heights at ±14 cm (1$\sigma$ level).

In Table 4, we can see the percentage of the GPS/leveling benchmarks whose adjusted residuals *h-H-N* (after a constant bias fit) fall within a specified range of geoid uncertainty. The agreement between EGM08 and GPS/H geoid heights is better than 10 cm for more than half of the total 1542 test points, whereas for the other GGMs the same consistency level is only reached at 18% (or less) of the test points. Furthermore, almost 85% of the test points give an agreement between the full-resolution EGM08 geoid and the GPS/leveling data that is better than 20 cm, compared to 36% (or less) in the case of all other global models that were tested.

**Table 4.** Percentage of the 1542 test points whose absolute values of their adjusted residuals $N^{GPS}$–$N$ *(after a least-squares constant bias fit)* are smaller than some typical geoid accuracy levels.

| | *< 2 cm* | *< 5 cm* | *< 10 cm* | *< 15 cm* | *< 20 cm* |
|---|---|---|---|---|---|
| EGM08 *($n_{max}$=2190)* | 13.3 % | 29.8 % | 53.5 % | 73.0 % | 84.6 % |
| EGM08 *($n_{max}$=360)* | 4.5 % | 11.6 % | 22.8 % | 32.7 % | 43.7 % |
| EIGEN-GL04C *($n_{max}$=360)* | 3.6 % | 9.3 % | 17.7 % | 27.5 % | 36.0 % |
| EIGEN-CG03C *($n_{max}$=360)* | 3.3 % | 8.3 % | 17.5 % | 26.5 % | 34.7 % |
| EIGEN-CG01C *($n_{max}$=360)* | 2.9 % | 7.8 % | 15.1 % | 23.0 % | 29.4 % |
| GGM02C *($n_{max}$=200)* | 2.9 % | 7.4 % | 15.0 % | 22.6 % | 30.2 % |
| EGM96 *($n_{max}$=360)* | 4.3 % | 9.8 % | 17.5 % | 27.7 % | 35.5 % |

The horizontal spatial variations of the (full-resolution) EGM08 residuals $N^{GPS}$-$N$ did not reveal any particular systematic pattern within the test network. Both their latitude-dependent and longitude-dependent scatter plots, as shown in Figures 2 and 3, are free of any sizeable north/south or east/west tilts over the Hellenic mainland. In other GGMs, however, some strong localized tilts and systematic oscillations can be identified in the $N^{GPS}$-$N$ residuals, mainly due to larger commission errors associated with their SHCs and significant omission errors involved in the recovery of the geoid signal (see Figures 2 and 3).

Our evaluation results have also confirmed that EGM08 performs exceedingly better than the other models over the mountainous parts of the Hellenic test network. A strong indication can be seen in the scatter plots of the pointwise residuals $N^{GPS}$-$N$ (after the constant bias fit) with respect to the orthometric heights of the corresponding GPS/leveling benchmarks (Figure 4). These plots reveal a *height-dependent bias* between the GGM and GPS/H geoid heights, which is considerably reduced in the case of EGM08. Apparently, the higher frequency content of the new model gives a better approximation for the terrain-dependent gravity field features over Greece, a fact that is visible from the comparative analysis of the scatter plots in Figure 4. The remaining height-dependent linear trend in the full-resolution EGM08 residuals $N^{GPS}$-$N$ (see Figure 4) is caused not only by commission/omission model errors, but it reflects also existing systematic problems in the orthometric heights of the tests points.

Further manifestation for the correlation of GGM and GPS/H geoid differences with the topographic height of the test points can be found in the color plots given in Figure 5. With the visual aid of the ETOPO2 digital elevation model, it is seen that larger values for the residuals $N^{GPS}$-$N$ occur mostly over geographical areas with strong topographic features. Note that the spatial distribution of the geoid height residuals for the full-resolution model EGM08 is depicted in two separate plots, each with a *different* color-scaling scheme. From the first of these plots, we can verify the overall improvement in the geoid representation over the Hellenic mountains that is achieved with EGM08, compared to the performance of previous GGMs over the same areas. The second scatter plot of the EGM08 geoid residuals $N^{GPS}$-$N$ (see lower left corner in Figure 5) reveals the remaining inconsistencies with the GPS/leveling data, which are caused by the commission/omission errors of the new model and other unknown systematic distortions in the orthometric heights at the test points.

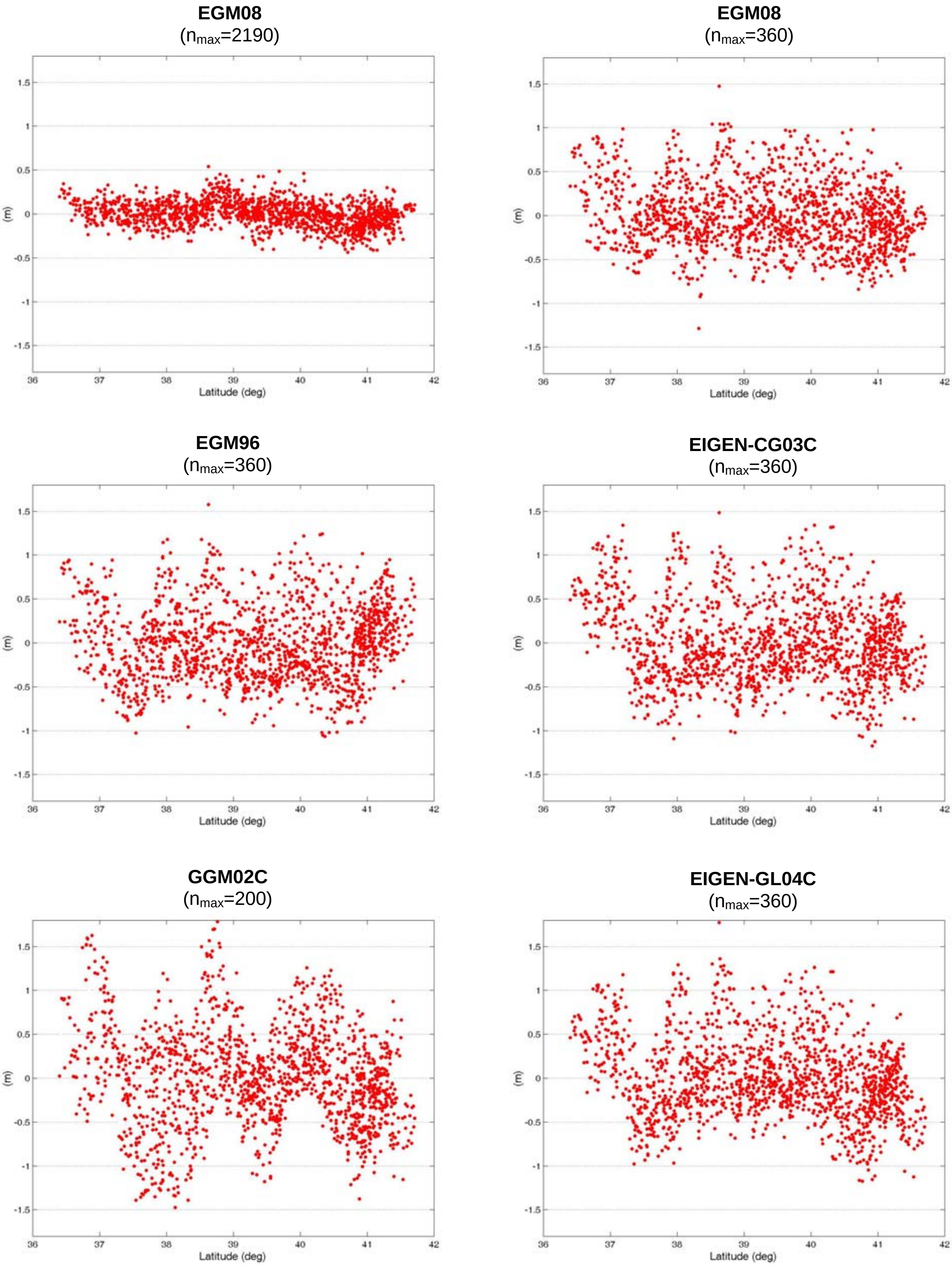


**Figure 2.** Latitude-dependent variations of the residuals $N^{GPS}$–$N$ *(after a least-squares constant bias fit)* at the 1542 GPS/leveling benchmarks.

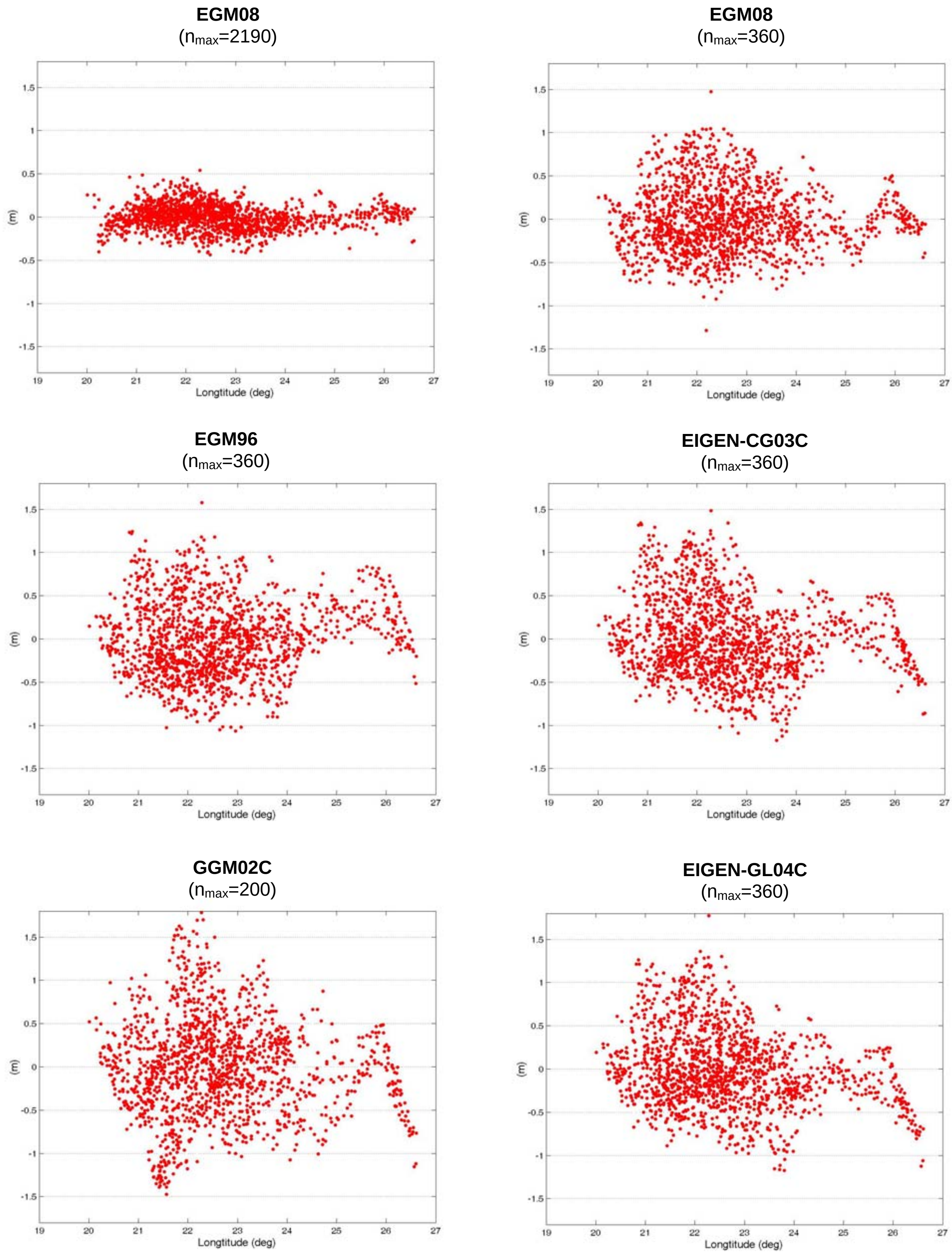


**Figure 3.** Longitude-dependent variations of the residuals $N^{GPS}–N$
*(after a least-squares constant bias fit)* at the 1542 GPS/leveling benchmarks.

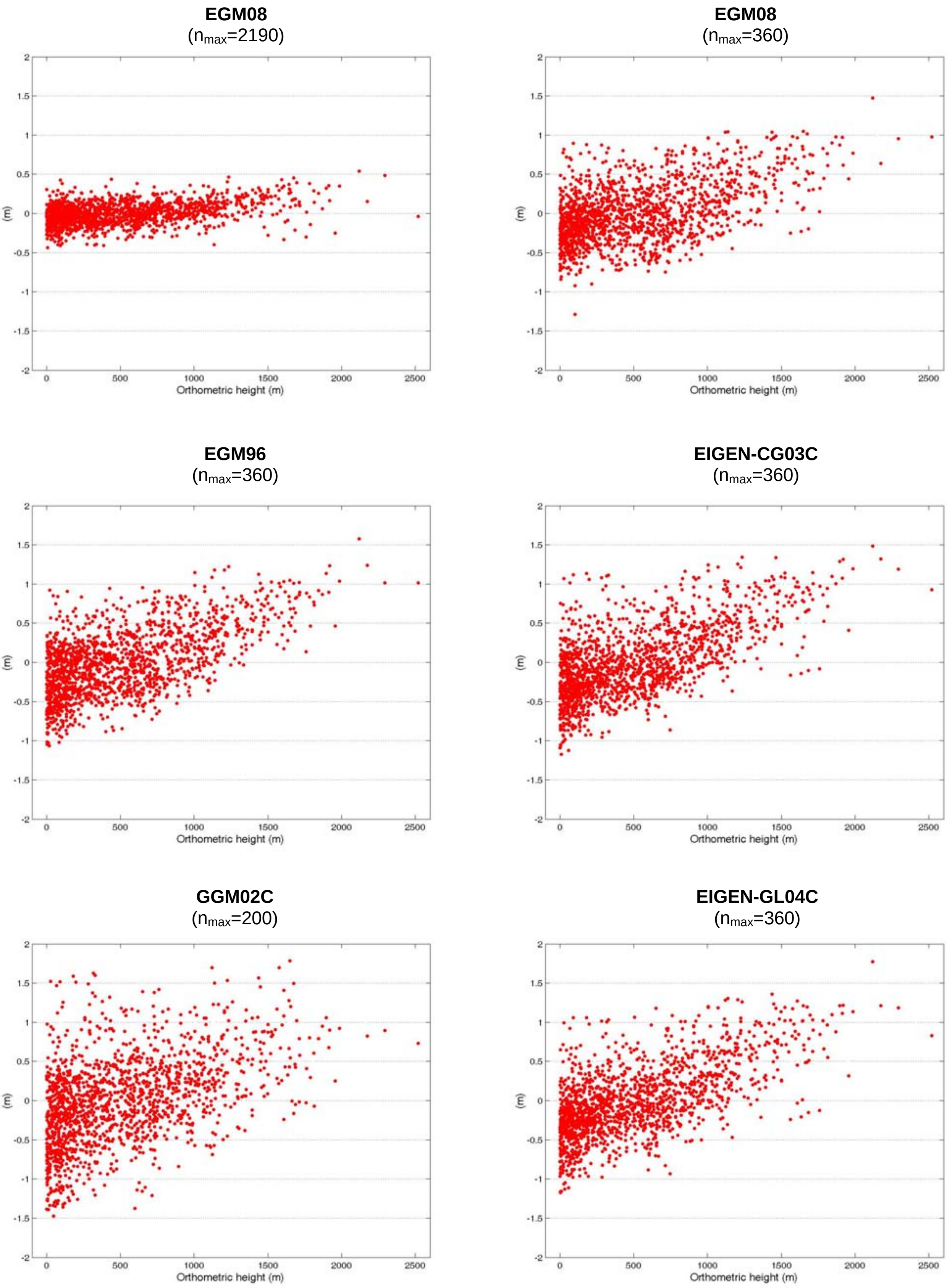


**Figure 4.** Height-dependent variations of the residuals $N^{GPS}–N$
*(after a least-squares constant bias fit)* at the 1542 GPS/leveling benchmarks.

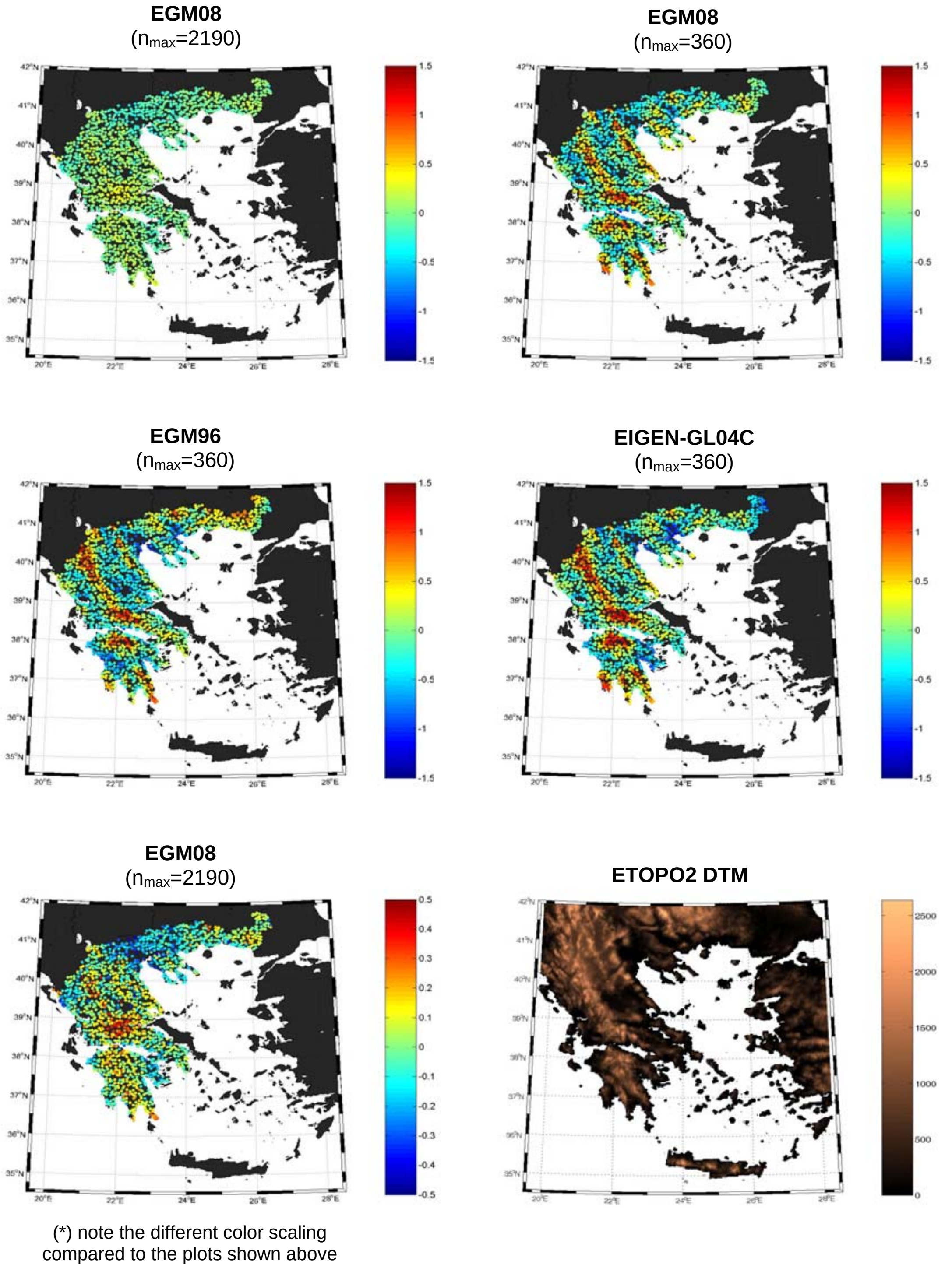


**Figure 5.** Colored scatter plots showing the geographical distribution of the differences $N^{GPS}$–$N$ *(after a least-squares constant bias fit)* at the 1542 GPS/leveling benchmarks.

## 4. POINTWISE EVALUATION TESTS WITH DIFFERENT PARAMETRIC MODELS

In addition to the evaluation results that were presented in the previous section, another set of numerical experiments has been carried out using a number of different parametric models for the least-squares adjustment of the differences $N^{GPS} - N$. The motivation for these additional tests was to investigate the fitting performance of some known linear models that are frequently used in geoid evaluation studies with heterogeneous height data, and to assess their feasibility in modeling the systematic discrepancies between the GGM and GPS/H geoid surfaces over the Hellenic mainland. Although these tests were implemented with all six GGMs that were initially selected for our study, only the results obtained with EGM08 and EGM96 will be presented herein due to space limitations.

The various parametric models that have been fitted to the original misclosures $h$-$H$-$N$ are given in Eqs. (5)-(10). *Model 1* uses a single constant-bias parametric term and it is actually the same model that was employed for all tests of the previous section. *Model 2* incorporates two additional parametric terms which correspond to an average north-south and east-west tilt between the GGM and GPS/H geoids. *Model 3* is the usual '4-parameter model' which geometrically corresponds to a 3D spatial shift and an approximate uniform scale change of the GGM's reference frame with respect to the underlying reference frame of the GPS heights (or vice versa). Finally, *models 4, 5 and 6* represent height-dependent linear corrector surfaces that constrain the relation among ellipsoidal, orthometric and geoidal heights in terms of the generalized equation

$$h - (1+\delta s_H)H - (1+\delta s_N)N = \mu \tag{4}$$

The above equation takes into consideration the fact that the spatial scale of the GPS heights does not necessarily conform with the spatial scale induced by the GGM geoid undulations and/or the inherent scale of the orthometric heights obtained from terrestrial leveling techniques. Moreover, the GGM geoid undulations and/or the local orthometric heights are often affected by errors that are correlated, to a certain degree, with the Earth's topography (see the results in Figures 4 and 5), a fact that can additionally justify the use of *model 4 or 6* for the optimal fitting between $N^{GPS}$ and $N$.

***Model 1***

$$h_i - H_i - N_i = \mu + v_i \tag{5}$$

***Model 2***

$$h_i - H_i - N_i = \mu + a(\varphi_i - \varphi_o) + b(\lambda_i - \lambda_o)\cos\varphi_i + v_i \tag{6}$$

***Model 3***

$$h_i - H_i - N_i = \mu + a\cos\varphi_i \cos\lambda_i + b\cos\varphi_i \sin\lambda_i + c\sin\varphi_i + v_i \tag{7}$$

***Model 4***

$$h_i - H_i - N_i = \mu + \delta s_H H_i + v_i \tag{8}$$

***Model 5***

$$h_i - H_i - N_i = \mu + \delta s_N N_i + v_i \tag{9}$$

***Model 6***

$$h_i - H_i - N_i = \mu + \delta s_H H_i + \delta s_N N_i + v_i \qquad (10)$$

***Remark.*** *A combination of the above models (e.g. the '4-parameter' or the 'bias and tilt' model merged with a height-dependent scaling term) may also be useful in practice, depending on the behavior of the actual data.*

The statistics of the adjusted residuals $\{v_i\}$ in the test network of 1542 Hellenic GPS/leveling benchmarks, after the least-squares fitting of the previous parametric models, are given in Tables 5 and 6 for the case of EGM96 and EGM08, respectively.

**Table 5.** Statistics of the differences $N^{GPS}$–$N$ for the ***EGM96*** geoid heights, after the least-squares fitting of various parametric models at the 1542 GPS/leveling benchmarks (units in m).

| | *Max* | *Min* | *σ* | *Bias (μ)* |
|---|---|---|---|---|
| ***Model 1*** | 1.577 | -1.063 | 0.423 | -0.446 |
| ***Model 2*** | 1.587 | -1.073 | 0.422 | -0.445 |
| ***Model 3*** | 1.681 | -1.097 | 0.411 | 303.983 |
| ***Model 4*** | 1.198 | -0.847 | 0.341 | -0.735 |
| ***Model 5*** | 1.572 | -1.053 | 0.423 | -0.381 |
| ***Model 6*** | 1.176 | -0.861 | 0.341 | -0.656 |

**Table 6.** Statistics of the differences $N^{GPS}$–$N$ for the ***EGM08*** geoid heights, after the least-squares fitting of various parametric models at the 1542 GPS/leveling benchmarks (units in m).

| | *Max* | *Min* | *σ* | *Bias (μ)* |
|---|---|---|---|---|
| ***Model 1*** | 0.542 | -0.437 | 0.142 | -0.377 |
| ***Model 2*** | 0.521 | -0.398 | 0.137 | -0.377 |
| ***Model 3*** | 0.522 | -0.398 | 0.137 | 3.479 |
| ***Model 4*** | 0.480 | -0.476 | 0.131 | -0.440 |
| ***Model 5*** | 0.528 | -0.442 | 0.135 | -0.109 |
| ***Model 6*** | 0.474 | -0.421 | 0.123 | -0.160 |

From the above results, it can be concluded that the low-order parametric models which are commonly used in the combined adjustment of GPS, geoid and leveled height data (*models 2* and *3*) do not offer any significant improvement for the overall fitting between the EGM08 geoid (or the EGM96 geoid) and the GPS/leveling heights over the Hellenic mainland. On the other hand, a purely height-dependent parametric model (*model 6*) enhances the statistical fit between the EGM08 and the EGM96 geoid with the GPS/leveling heights by 2 cm and 8 cm, respectively (i.e. compared to the performance of the bias-only *model 1*). The improvement in the sigma values obtained from *models 4 and 6* should be attributed to the elimination of the linear correlation trend that was previously identified (see Figure 4) between the misclosures *h-H-N* and the orthometric heights of the test points.

Note that all alternative models which are tested in this section include a common parametric term in the form of a *single constant bias*. However, the various estimates of the common bias parameter $\mu$, as obtained from the least-squares adjustment of each model, exhibit significant variations among each other (see last column in Tables 5 and 6). Specifically, the estimated bias between $N^{GPS}$ and $N$ which is computed from the usual '4-parameter' model appears to be highly inconsistent with respect to the corresponding estimates from the other parametric models. This is not surprising since the intrinsic role of the bias $\mu$ in *model 3* is not to represent the average *spatial offset* between the GGM and the GPS/H geoids, as it happens for example in the case of *model 1*. In fact, the three additional parametric terms in *model 3* are the ones that absorb the systematic part of the differences $N^{GPS} - N$ in the form of a three-dimensional spatial shift ($a \rightarrow t_x$, $b \rightarrow t_y$, $c \rightarrow t_z$), leaving to the fourth bias parameter $\mu$ the role of a 'scale-change' effect.

At this point, it is perhaps instructive to recall the linearized transformation formula for geoid heights between two parallel geodetic reference frames (see, e.g., Kotsakis 2008)

$$N_i' - N_i = (\mathrm{a}w_i + N_i)\delta s + t_x \cos\varphi_i \cos\lambda_i + t_y \cos\varphi_i \sin\lambda_i + t_z \sin\varphi_i \tag{11}$$

where a denotes the semi-major axis of the common reference ellipsoid, $\delta s$ is the differential scale change between the underlying frames, and $w_i$ corresponds to the auxiliary unitless term $(1 - e^2 \sin^2 \varphi_i)^{1/2}$ that is approximately equal to 1 (i.e. the squared eccentricity of the reference ellipsoid is $e^2 \approx 0.0067$). The above formula conveys, in the language of geodetic datum transformation, the basic geometric principles of the '4-parameter' model that is frequently employed for the optimal fitting of GPS, geoid and leveled height data. Given the analytic expression in Eq. (11), the constant bias $\mu$ that appears in the formulation of *model 3* emulates the effect of a mean spatial re-scaling rather than a mean spatial offset between two different geoid realizations.

Although less inconsistent with each other, the estimates of the bias parameter $\mu$ from the other parametric models show dm-level fluctuations in their values. It should be noted though that the inclusion of additional spatial tilts for the fitting between $N^{GPS}$ and $N$ does not distort the initial estimate of $\mu$ that was obtained from *model 1* over the Hellenic mainland. On the other hand, the use of height-dependent scaling terms (*models 4*, *5* and *6*) affects considerably the final estimates of the bias parameter $\mu$, as it can be easily verified from the results in Tables 5 and 6.

All in all, the problem of obtaining a realistic estimate for the average spatial offset between a local vertical datum (e.g. HVD in our case) and a GGM geoid seems to have a strong dependence on the parametric model that is used for the adjustment of heterogeneous height data over a test network of GPS/leveling benchmarks. Since there exist strong theoretical and practical arguments that can be stated in favor of the generalized constraint in Eq. (4), the use of the simple *model 1* is not necessarily the safest choice for estimating the average spatial offset between GGM and GPS/H geoids over a regional network. In view of the frequent absence (or even ignorance) of a complete and reliable stochastic error model for the properly weighted adjustment of the differences $N^{GPS} - N$, a clear geometrical interpretation of the estimated bias $\mu$ is not always a straightforward task in GGM evaluation studies.

## 5. BASELINE EVALUATION TESTS

An additional set of evaluation tests was also performed through the comparison of GGM and GPS/H *geoid slopes* over the Hellenic network of 1542 GPS/leveling benchmarks. For all baselines formed within this network, the following differences of relative geoid undulations were determined

$$\Delta N_{ij}^{GPS} - \Delta N_{ij} = (h_j - H_j - h_i + H_i) - (N_j - N_i) \tag{12}$$

Note that the computation of the above differences took place *after* the implementation of a least-squares bias/tilt fit between the pointwise values of the GGM and GPS/H geoid heights.

Depending on the actual baseline length, the residual values from Eq. (12) were grouped into various spherical-distance classes and their statistics were then evaluated within each class. Given the actual coverage and spatial density of the GPS/leveling benchmarks in our test network, baselines with length from 2 km up to 600 km were considered for this evaluation scheme. The statistics of the differences between the GGM and GPS/H relative geoid heights, for five selected baseline classes, are given in the following tables.

**Table 7.** Statistics of the differences between GGM and GPS/H relative geoid heights for baselines with length < ***3 km*** (number of baselines: 47, units in m).

| | *Max* | *Min* | $\sigma$ | *Bias* |
|---|---|---|---|---|
| EGM08 *($n_{max}$=2190)* | 0.142 | -0.156 | 0.058 | -0.009 |
| EGM08 *($n_{max}$=360)* | 0.140 | -0.206 | 0.080 | -0.018 |
| EIGEN-GL04C *($n_{max}$=360)* | 0.156 | -0.200 | 0.087 | -0.015 |
| EIGEN-CG03C *($n_{max}$=360)* | 0.148 | -0.205 | 0.087 | -0.016 |
| EIGEN-CG01C *($n_{max}$=360)* | 0.152 | -0.207 | 0.087 | -0.016 |
| GGM02C *($n_{max}$=200)* | 0.137 | -0.230 | 0.081 | -0.021 |
| EGM96 *($n_{max}$=360)* | 0.136 | -0.199 | 0.081 | -0.014 |

**Table 8.** Statistics of the differences between GGM and GPS/H relative geoid heights for baselines with length < ***5 km*** (number of baselines: 289, units in m).

| | *Max* | *Min* | $\sigma$ | *Bias* |
|---|---|---|---|---|
| EGM08 *($n_{max}$=2190)* | 0.643 | -0.474 | 0.111 | 0.006 |
| EGM08 *($n_{max}$=360)* | 0.648 | -0.534 | 0.154 | 0.003 |
| EIGEN-GL04C *($n_{max}$=360)* | 0.649 | -0.542 | 0.155 | 0.005 |
| EIGEN-CG03C *($n_{max}$=360)* | 0.643 | -0.540 | 0.155 | 0.005 |
| EIGEN-CG01C *($n_{max}$=360)* | 0.640 | -0.536 | 0.156 | 0.005 |
| GGM02C *($n_{max}$=200)* | 0.685 | -0.571 | 0.162 | 0.003 |
| EGM96 *($n_{max}$=360)* | 0.643 | -0.553 | 0.154 | 0.005 |

**Table 9.** Statistics of the differences between GGM and GPS/H relative geoid heights for baselines with length ***5-10 km*** (number of baselines: 2119, units in m).

| | *Max* | *Min* | $\sigma$ | *Bias* |
|---|---|---|---|---|
| EGM08 *($n_{max}$=2190)* | 0.465 | -0.629 | 0.125 | 0.001 |
| EGM08 *($n_{max}$=360)* | 1.022 | -1.044 | 0.248 | -0.004 |
| EIGEN-GL04C *($n_{max}$=360)* | 0.983 | -0.988 | 0.251 | -0.000 |
| EIGEN-CG03C *($n_{max}$=360)* | 0.971 | -1.026 | 0.251 | -0.001 |
| EIGEN-CG01C *($n_{max}$=360)* | 0.976 | -1.039 | 0.252 | -0.002 |
| GGM02C *($n_{max}$=200)* | 0.967 | -0.991 | 0.264 | 0.002 |
| EGM96 *($n_{max}$=360)* | 0.963 | -1.002 | 0.251 | 0.003 |

**Table 10.** Statistics of the differences between GGM and GPS/H relative geoid heights for baselines with length ***10-50 km*** (number of baselines: 56575, units in m).

| | *Max* | *Min* | $\sigma$ | *Bias* |
|---|---|---|---|---|
| EGM08 *($n_{max}$=2190)* | 0.859 | -0.781 | 0.164 | -0.001 |
| EGM08 *($n_{max}$=360)* | 2.778 | -2.417 | 0.514 | -0.012 |
| EIGEN-GL04C *($n_{max}$=360)* | 2.480 | -2.430 | 0.552 | -0.019 |
| EIGEN-CG03C *($n_{max}$=360)* | 2.335 | -2.488 | 0.550 | -0.021 |
| EIGEN-CG01C *($n_{max}$=360)* | 2.335 | -2.445 | 0.555 | -0.021 |
| GGM02C *($n_{max}$=200)* | 3.221 | -2.760 | 0.627 | -0.012 |
| EGM96 *($n_{max}$=360)* | 2.532 | -2.393 | 0.542 | -0.013 |

**Table 11.** Statistics of the differences between GGM and GPS/H relative geoid heights for baselines with length ***50-100 km*** (number of baselines: 135970, units in m).

| | *Max* | *Min* | $\sigma$ | *Bias* |
|---|---|---|---|---|
| EGM08 *($n_{max}$=2190)* | 0.891 | -0.881 | 0.189 | -0.003 |
| EGM08 *($n_{max}$=360)* | 2.332 | -2.282 | 0.552 | -0.013 |
| EIGEN-GL04C *($n_{max}$=360)* | 2.410 | -2.773 | 0.658 | -0.041 |
| EIGEN-CG03C *($n_{max}$=360)* | 2.172 | -2.568 | 0.651 | -0.043 |
| EIGEN-CG01C *($n_{max}$=360)* | 2.356 | -2.600 | 0.668 | -0.037 |
| GGM02C *($n_{max}$=200)* | 3.043 | -3.611 | 0.834 | -0.067 |
| EGM96 *($n_{max}$=360)* | 2.226 | -2.480 | 0.623 | -0.028 |

As seen from the results in Tables 7 through 11, the full-resolution EGM08 model performs consistently better than all other GGMs over all baseline classes. The improvement becomes more pronounced as the baseline length increases, indicating the significant contribution of the EGM08 high-degree harmonics ($n$ > 360) for the slope representation of the Hellenic geoid over baselines 5-100 km. For example, the resultant $\sigma$ values of the differences $\Delta N^{GPS} - \Delta N$ are reduced by a factor of 1.4 for baselines <5 km, by a factor of 2 for baselines 5-10 km, and by a factor of about 3.5 for baselines 10-100 km (compared to the performance of EGM96 and other EIGEN-type models).

It is also interesting to observe the considerable bias in the geoid slope residuals $\Delta N^{GPS} - \Delta N$ obtained from all tested GGMs (except from the full-resolution EGM08 model) for baselines 10-100 km. This result should be attributed to existing systematic errors in the medium-wavelength SHCs of the tested GGMs and additional omission errors in the pre-EGM08 models, which produce an apparent scale difference between GGM and GPS/H relative geoid undulations for the aforementioned baseline range.

The overall behaviour of the sigma values for the differences between GGM and GPS/H geoid slopes is shown in Figure 6, over all baseline classes that were considered in our tests. The remarkable improvement in the relative geoid accuracy from the EGM08 model is clearly visible, indicating an $\Delta N$-consistency level with the external GPS/leveling data that varies from ±6 cm to ±20 cm (1$\sigma$ level).

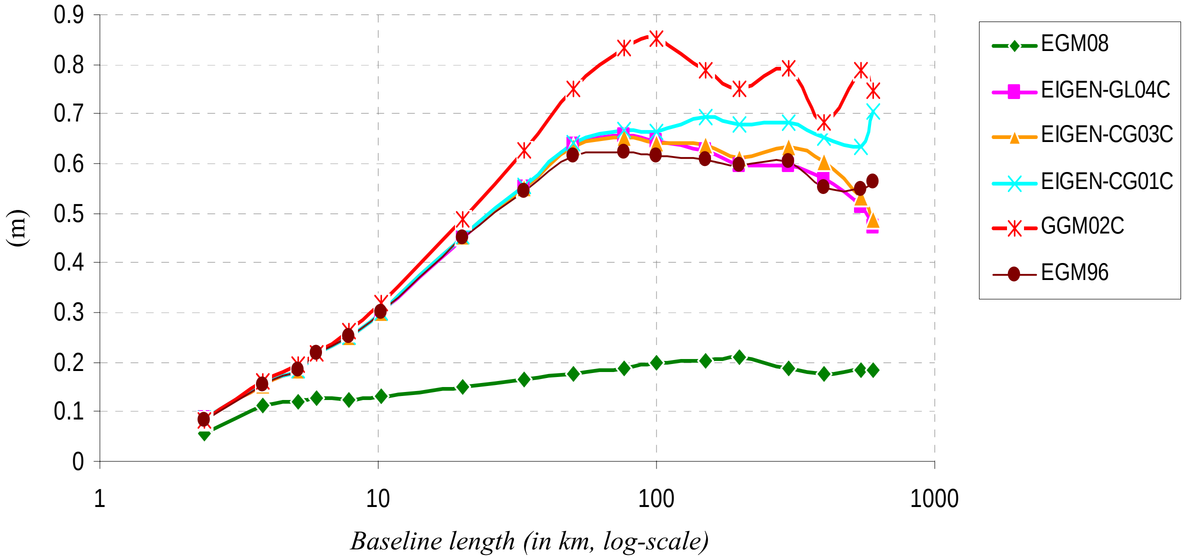


**Figure 6.** Std of the differences $\Delta N_{ij}^{GPS} - \Delta N_{ij}$ in the test network of 1542 GPS/leveling benchmarks, as a function of the baseline length.

Focusing on the geoid-slope evaluation results for short baselines (up to 30 km) can give us an indication for the expected accuracy in GPS/leveling projects when using an EGM08 reference geoid model over Greece. Our preliminary analysis in the test network showed that the agreement between the height differences $\Delta H_{ij}$ computed: **(a)** directly from the known orthometric heights at the GPS/levelling benchmarks and **(b)** indirectly from the GPS/EGM08 ellipsoidal and geoid heights, could be approximated by the statistical error model $\sigma_{\Delta H} = \sigma_o L^{1/2}$ with the a-priori sigma factor $\sigma_o$ ranging between 3-5 cm/km (for baseline length $L$<30 km). Although such a performance cannot satisfy mm-level accuracy requirements for vertical positioning (which are 'easily' achievable through spirit leveling techniques), it nevertheless provides a major step forward that can successfully accommodate a variety of engineering and surveying applications. Note that the corresponding performance of EGM96 in our test network is described by a relative accuracy factor of $\sigma_o \approx 9$ cm/km for baselines <30 km.

## 6. Conclusions

The results of our evaluation tests have revealed the superiority of EGM08 over all existing mixed GGMs for the area of Greece. The new model outperforms the other tested GGMs at the 1542 Hellenic GPS/leveling benchmarks and it improves the statistical fit with the Hellenic GPS/H geoid by approximately 30 cm (or more)! The pointwise agreement among ellipsoidal, orthometric and EGM08-based geoid heights is at ±14 cm ($1\sigma$ level), reflecting mainly the regional effects of the commission errors in the model's SHCs, as well as other local distortions in the HVD orthometric heights at the control points.

In terms of relative geoid accuracy, EGM08 shows a rather stable performance for the standard deviation of the slope residuals $\Delta N^{GPS} - \Delta N$ over all baseline lengths that were considered in our study. Compared to other tested GGMs whose relative geoid accuracy decreases continuously over baselines 5-100 km (estimated values for $\sigma_{\Delta N}$ reach up to 60 cm), the full-resolution EGM08 model gives a more balanced behavior with the corresponding values of $\sigma_{\Delta N}$ not exceeding 20 cm, even for baselines up to 600 km.

In conclusion, the results presented herein provide a promising testament for the future use of EGM08 in geodetic applications over the Hellenic mainland. However, in view of its possible forthcoming implementation in GPS-based leveling projects throughout Greece (in conjunction with the HEPOS system), a more detailed analysis with additional interpolation methods and spatial 'corrector surfaces' for modeling the differences $N^{GPS} - N$ or $\Delta N^{GPS} - \Delta N$ is required to achieve cm-level consistency for the transformation between GPS/EGM08 and HVD orthometric heights.

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