WEIGHTING OF CODE DOUBLE-DIFFERENCES BY MEANS OF SNR: THEORY AND VALIDATION TESTS

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„Die meiste Energie verbraucht der Mensch mit der Lösung von Problemen, die niemals auftreten werden", Somerset Maugham.

ABSTRACT

The improvement of the technology of the GPS-receivers within the last years led to sub-meter precision of the code measurements. This allowed a code-based differential kinematic positioning with accuracy of a few meters, which is sufficient for a variety of applications. However, for special applications, like the CAT III precision approach and landing of an aircraft, there is a need to enhance the accuracy of code-Differential GPS (DGPS).

The paper describes a method, that improves the code-DGPS solution. The method is based on a realistic computation of the covariance matrix of the observations which is used in the least-squares adjustment. The proposed algorithm uses for the computation of the weighting matrix the signal-to-noise ratio (SNR) given by the receiver. In the first part of the paper the theoritical background of the ordinary and proposed weighting is given. In the second part a summary of extensive validation tests comparing the results of the two processing methods is given.

1. Introduction

Differential GPS is a very powerful system for positioning and navigation. In a kinematic environment code-DGPS is capable of yielding accuracies of 1-5 m. A higher accuracy, e.g. 0.5 m, cannot be guaranteed by code-DGPS. If such an accuracy level is needed, the phase-GPS technique can be used, which leads to centimeter-precision. This technique however is susceptible to cycle-slips and requires the resolution of the ambiguities, which is (especially in a real-time environment) a difficult and time-consuming task.

In order to overcome these limitations, techniques have been developed for enhancing the potential of the code solution by use of the phase measurements without ambiguity resolution. The most known method is the so-called phase-smoothing of the codes [Lachapelle et al., 1986]. This paper describes an algorithm, that improves the accuracy of code-DGPS without using the phase measurements. The accuracy improvement in comparison to the usual processing of the code observations is achieved by the fact that the code double-differences (DD) are not equally weighted. Besides the improvement of the accuracy another important advantage of this method is that observations to satellites with low elevation (< 15°) can be used, which improves furthermore the results.

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2. Theoretical background

Generally, only few attension is payed on the fact that the precision of the GPS-observations depends on many factors. In this chapter will be shown up to what extend the precision of the individual code-DDs of one epoch can vary. In addition, the ordinary and the proposed method for weighting the DDs are described.

2.1 Investigation of the precision of the code-DDs

It is known that the precision of the code- or phase-measurements depends strongly on the type of the receiver (e.g. geodetic or navigation receiver) and the tracking method (squaring, L1/L2-cross-correlation, Z-tracking etc.) [Breuer et al., 1993]. However, for a given receiver the precision of the observations depends also on several factors. The most important of them are the satellite elevation, the atmospheric conditions and interfering signals.

One possibillity to estimate the noise level of a DD is by means of the second order time differences of this DD. This method is extensively discussed in [Gianniou, 1995] and [Gianniou, 1996]. Within a short time span the geometric (computed) DDs can be approximated quite well by a polynomial of second degree. By building the second order time differences the linear and quadratic term cancel out. If the data are sampled at a high rate, the variation in time of the time differences discribes the noise level of the observable. In order to take into account instabilities of the sampling rate due to limitations of the receiver clock, the first order time differences are normalized by the time interval between the two epochs. The normalized first and second order time differences of the DDs ($\nabla\Delta$) read [Gianniou, 1996]:

$$\delta_n \nabla\Delta\left(t_{k+1}\right) = \frac{\nabla\Delta\left(t_{k+1}\right) - \nabla\Delta\left(t_k\right)}{t_{k+1} - t_k} \tag{1}$$

and

$$\delta_n^2 \nabla\Delta\left(t_{k+2}\right) = \delta\nabla\Delta\left(t_{k+2}\right) - \delta\nabla\Delta\left(t_{k+1}\right) = \frac{\nabla\Delta\left(t_{k+2}\right) - \nabla\Delta\left(t_{k+1}\right)}{t_{k+2} - t_{k+1}} - \frac{\nabla\Delta\left(t_{k+1}\right) - \nabla\Delta\left(t_k\right)}{t_{k+1} - t_k}. \tag{2}$$

The figures 1a and 1b show the normalized second order time differences of two C/A-code DDs. The data are collected with Trimble 4000SSE receivers on a short baseline. The comparison of the two figures makes clear, how variable the precision between the observations of one epoch can be.

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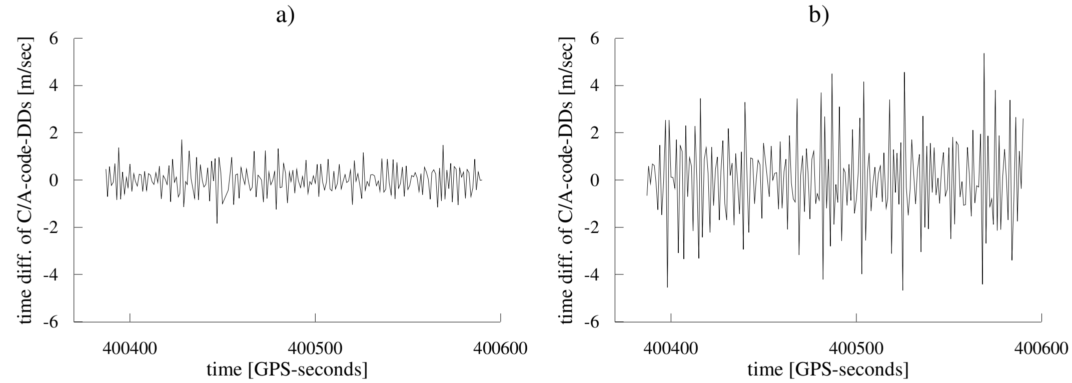

Figure 1: Noise level of two code DDs refering to the same baseline occupation: a) DD betwenn PRN 9 and 5 and b) between PRN 9 and 26.

2.2 Ordinary weighting of the code-DDs

Usually, the code- or phase-DDs are adjusted in a least squares sence assuming that all observations are of the same precision. In other words all diagonal elements of the weight matrix are equal. The non-diagonal elements are not zero in order to take into account the correlations between the DDs. The corresponding weight matrix for adjusting the data of one epoch is given by [Hofmann-Wellenhof et al., 1994]:

$$P = \frac{1}{2\sigma^2}\frac{1}{n+1}\begin{bmatrix} n & -1 & -1 & \cdots \\ -1 & n & -1 & \cdots \\ -1 & & \ddots & \\ \vdots & \cdots & & n \end{bmatrix} \tag{3}$$

where n is the number of the DDs in the current epoch and σ^2 is the variance of the (undifferenced) code observations. This matrix can also be used for adjusting phase-DDs, given that the ambiguities are already solved.

2.3 Weighting of the code-DDs by means of signal-to-noise ratio (SNR)

The figures 1a and 1b showed how variable the noise level of the observations of one epoch can be. A way to estimate the precision of a code-DD is by means of the standard deviation of the normalized second order time differences (i.e. the standard deviation of the time series in figures 1a-b) [Gianniou, 1995]. However, this method is limited to off-line processing of static measurements. Furthermore, it requires a high sampling rate.

In order to overcome these limitations, a method has been developed, where the computation of the weight matrix is based on the signal-to-noise ratio. Roughly

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speaking, the SNR expresses the power of the signal relative to that of the noise. Thus, the higher the SNR the better the measurement. The relation between the SNR and the standard deviation of the code-measurement in the delay-lock loop (DLL) is given by [Ward, 1994]:

$$\sigma_{DLL} = \sqrt{\frac{FB_n}{c/n_0}\left[1+\frac{2}{T\,c/n_0}\right]}\;\lambda_C \tag{4}$$

with F the DLL discriminator correlator factor, B_n the code loop noise bandwidth, c/n the SNR expressed as a ratio, T the predetection integration time and λ_C the "wave-length" of the code. However, the relation between σ and SNR depends on the tracking method and the formula (4) is not unique. Other similar formulas can be found in [van Dierendonck et al., 1992] and [Sideris et al., 1992].

The relationship between the measurement precision and the SNR is shown in figures 2a and 2b. The first figure shows the L1-SNR for the satellite PRN 6 as given by a Trimble 4000SSE receiver. The second figure illustrates the normalized second order time differences of the L1-code-measurement to the same satellite. It can clearly be seen that reduction of SNR corresponds to increased noise level of the observation.

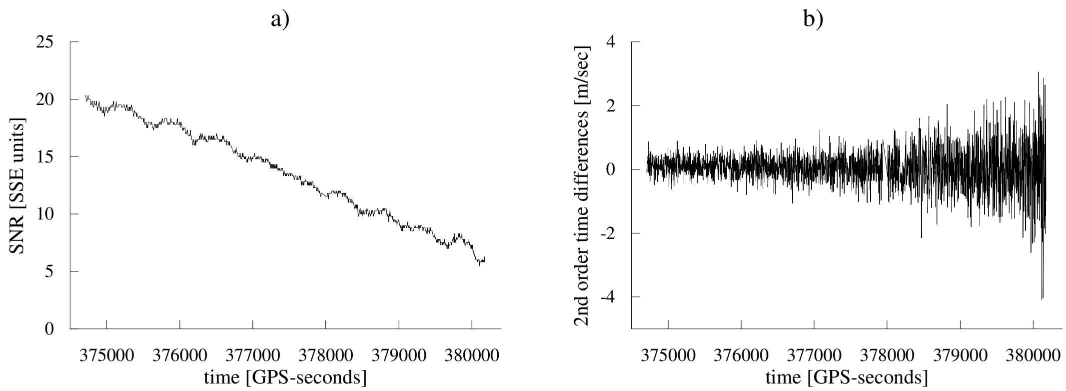

Figure 2: Relation between the SNR and the noise level of the code-measurement.

The main reason why the SNR in figure 2a becomes lower is that the elevation of the satellite above the horizon of the observation site becomes lower (40^o - 10^o). By decreasing elevation the satellite signal travels a longer distance and - more important - its path through the atmosphere becomes longer. As a consequence more signal power is getting lost and in addition more noise is being added to the transmitted signal. In other words the lower the satellite elevation the worse the observation.

The relationship between elevation and measurement precision has been investigated by Euler and Goad [1991]. In the same reference an empirical function connecting these two quantities is given. This function is working quite well, but it does not take into account other parameters that have an influence on the measurement precision.

The method described here uses instead of the elevation the signal-to-noise ratio for the estimation of the measurement noise. The SNR is influenced (besides by the elevation) also by a lot of occasional and variable aspects. This can be understood by the equation that gives the (unjammed) SNR [Ward, 1994]:

$$SNR = Sr + Ga - 10\,log\left(kT_o\right) - Nf - L \tag{5}$$

where *Sr* the received GPS signal power, *Ga* the antenna gain towards satellite, *k* the Boltzmann's constant, T_o the thermal noise reference temperature, *Nf* the noise figure of receiver including antenna and cable losses and *L* the implementation loss plus A/D converter loss. In case of interferences the SNR is being reduced and the so-called equivalent SNR should be used in the formula (4). The use of SNR for the estimation of the measurement precision has the advantage of considering parameters that depend on the receiver design (*L*, *Nf*, *Ga*) the transmitted signal power and the transmition loss (both affecting *Sr*) and environmental parameters like interferences.

The method described here uses the SNR for the computation of the measurement noise but not by means of the formula (4). The reasons for that are that this formula is not unique and that the involved parameters (*Bn*, *T*) are not always given by the manifuctures. Furthermore, some parameters are not constant depending on the observation mode (static or kinematic). In order to overcome these difficulties, an empirical function has been developed giving the measurement noise as a function of SNR. The formula is for use with data collected with Trimble SSE and SSi receivers and geodetic antennas. For developing this function, a differential measurement on a short baseline has been used. Nine satellites have been observed with Trimble SSE receivers. Their elevation angles had had a good distribution between 0^o and 85^o. The noise level of the (undiffereced) observations is computed by building the second order time differences of the DDs. This procedure is explained in detail in [Gianniou, 1996]. In figure 3 the estimated standard deviation of each observation is plotted against the corresponding SNR. The solid line is the computed empirical function, which reads:

$$\sigma_{CODE} = 0.09 + 0.975e^{-0.11SNR} \ . \tag{6}$$

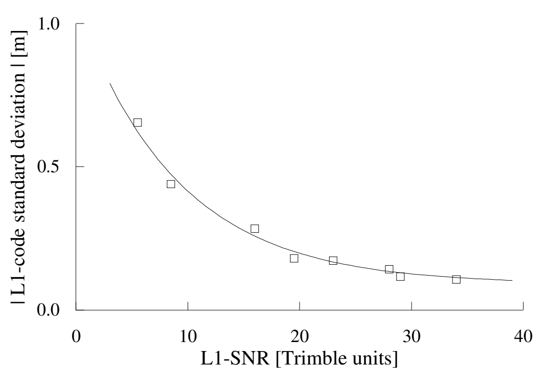

Figure 3: Computed empirical function connecting noise of code measurement and SNR.

In an epochwise adjustment first the variance of the undifferenced code measurements is computed using the formula (6). Then, applying error propagation, the covariance matrix of the DDs is computed. By inversion of this matrix we obtaine the weight matrix in which all non-diagonal elements are equal. Between the diagonal elements considerable differences have often been encountered. The following matrix is an example for the covariance matrix of the DDs.

$$Cov\left(\nabla\Delta_{Code}\right) = \begin{bmatrix} \mathit{0.08} & & & & \mathrm{K} & \mathit{0.07} \\ & \mathit{0.21} & & & \mathrm{N} & \mathrm{M} \\ & & \mathit{0.80} & & & \\ & & & \mathit{0.82} & & \\ \mathrm{M} & \mathrm{N} & & & \mathit{0.85} & \\ \mathit{0.07} & \mathrm{K} & & & & \mathit{0.92} \end{bmatrix} \qquad (7)$$

Looking at the figures 1a and 1b one could understand why the use of a weight matrix like that in (7) would lead to much better results than the use of the weight matrix in (3). This will be demonstrated by the analysis of field data in the next chapter. Some limitations of the proposed method as well as a comparison between the weighting by means of elevation an by means of SNR are placed in chapter 4.

3 Experimental results

Besides the ipmprovement in the estimation of the coordinates the method proposed here has the advantage that observations to satellites with low elevation can be used. These measurements are quite noisy and are not used when the observations are uniformly weighted. Obviously, the optimal elevation cut-off angle depends on the weighting method. In order to find the optimal limit for both cases and to compare the results a data set consisting of 27 baselines has been analysed [Biadse, 1996]. Both static and kinematic differential measurements over distances from 100 m to 11 km under different circumstances (satellite constellation, free sky view, near trees or power lines etc.) have been conducted and processed using both weighting methods. For all measurements Trimble 4000 SSi receivers and antennas with ground planes have been used.

3.1 Weighting by means of SNR and accuracy improvement

In order to show the accuracy improvement that can be achieved by the proposed weighting method, one typical example is given here. The kinematic measurement was conducted in Oktober 1995. Three Trimble SSi receivers have been used (two reference stations and one rover receiver). The rover receiver was installed on a van. The elevation mask was set to 5^{o}. In figure 4a the error in X-coordinate for the usual and proposed weighting is given.The difference between maximum and minimum error for the whole observation time for both cases is shown in figure 4b. In order to investigate the influence of the baseline length on the efficiency of the proposed method, the data have been processed using both the near (1 km) and the far (7 km) reference station.

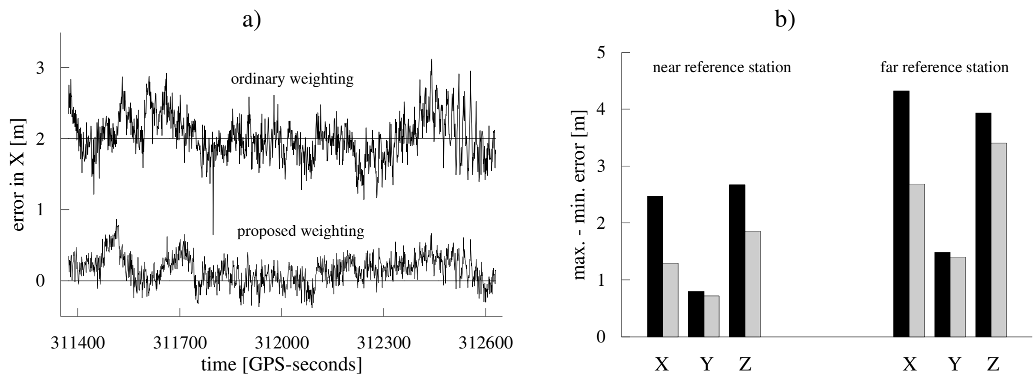

Figure 4: Ordinary versus proposed weighting: a) Error in X-coordinate (upper curve is shifted by 2 meters), b) Difference between max. and min. error (dark bars correspond to the ordinary and light bars to the proposed weighting).

3.2 Estimation of the optimal cut-off angles

In order to find the optimal cut-off angles for both weighting methods, each of the 27 baselines mentioned at the beginning of chapter 3 has been processed several times. In the first processing all observed satellites have been used. In the second processing the lowest satellite has been excluded, in the third the two lowest satellites and so on for all satellites below 20^{o}. For the computation of the coordinate errors of the code solution the phase solution has been used as reference. The figure 5 illustrates the standard deviation of the X-coordinate for two typical baseline examples for the ordinary and the proposed weighting. The standard deviation is computed from the time serie of the X-coordinate obtained from an epochwise adjustment. Clearly, the satellites below 20^{o} do

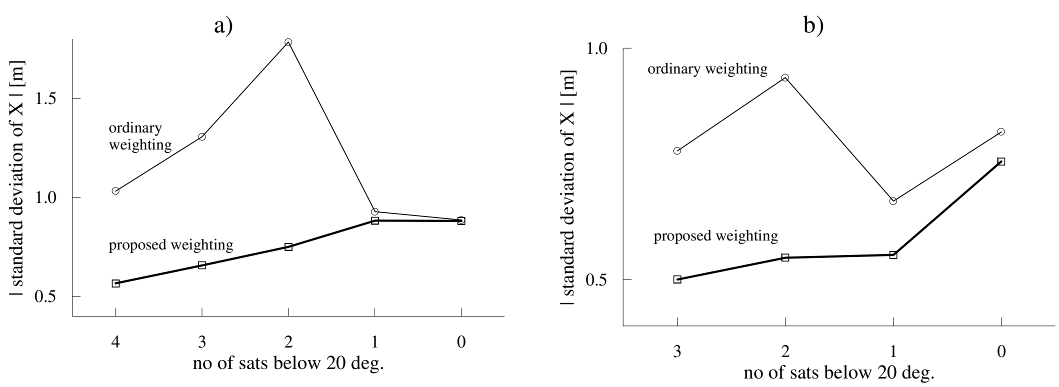

Figure 5: Standard deviation for ordinary and proposed weighting in dependance on the number of used satellites below 20^o.

not have a predictable influence on the precision of the estimated coordinates when using the ordinary weighting. On the contrary, these satellites improve the solution when the proposed weighting method is used. After consideration of the results for all 27 baselines it could be concluded that the optimal elevation masks for the usual and proposed weighting are 15^o and 5^o respectively. These values refer to measurements with Trimble SSE and SSi receivers. They can vary by some degrees depending on what is more important in an appllication: the achieved accuracy or the availability of as much satellites as possible.

4 Discussion

The normalized second order time differences have been used to investigate the noise level of the code-DDs and to estimate the empirical function (6). The time differences can give valuable information about the measurement precision if two conditions are fulfiled: The data must be sampled at a high rate (> 0.2 Hz) and the DDs should not be affected by coloured noise, because such effects cancel out when building time differences. For the Trimble SSE and SSi receivers the second condition is found out to be fulfiled [Gianniou, 1996].

The weighting by means of SNR has been extensively tested with static and kinematic measurements (antenna installed on van and on airplanes). It can be expected that these test measurements were affected by multipath. It is known that multipath affects the SNR. In static measurements this can easily be seen by its periodic variations. The detection of multipath in a kinematic environment is much more difficult. Generally, when the SNR is increased due to multipath, this does not mean that the measurement is better (according to formulas (4) and (6)). The problem originates from the general difficulty in distinguishing between signal and noise. The reflected signal that overlays with the direkt one is also a GPS-signal. It is simply time-delayed. If this delay is not long, the receiver cannot distinguish between the desired and the undesired component. This situation can also occur in exceptional cases of strong interferences [Gianniou, 1996]. The efficiency of the proposed method has not been tested in a marine environment where strong multipath effects are known to occur. However, for ordinary airborne applications the SNR of the Trimble SSi receivers is not seriously affected by multipath.

The improvement of the accuracy of the code-DD coordinates that can be achieved by the SNR-weighting is shown in figures 3 and 4. However, the improvement of the code solution is important not only for code DGPS but also for carrier phase DGPS, especially in real-time. The reason for that is that good code coordinates allow for a smaller searching space when fixing the float ambiguities. An extensive discussion of this point can be found in [Gianniou and Groten, 1996].

5 Conclusions

The computation of normalized second order time differences of the code-DDs proved to be an adequate way for the investigation of the noise level of the DDs.
Based on this method an empirical function connecting the SNR and he measurement precision has been computed.

The use of this empirical function for the computation of the weight matrix in an epochwise adjustment of code-DDs leads to an increased accuracy in comparison to the ordinary weighting, where all observations are equally weighted.

Another advantage of the proposed method is that the noisy observations to satellites below 20^o can be used. This leads to a further accuracy improvement because the redundancy of the system and the geometry of the observations become better. Of coarse, the redundancy and the geometry become better also if the observations are equally weighted but in that case the advantages are mitigated by the introduction of noise into the system (i.e. noisy measurements that have the same contribution to the results as all other ones).

The accuracy improvement achieved by the proposed method is from particular importance for kinematic applications where the coordinates of each point are obtained from the observations of only one epoch. Coordinate error reduction up to 1m has been encounted.

The second advantage of the proposed method, namely the utilization of satellites with low elevation, is particularly important for airborne and marine applications where no obstacles disturb the reception of the signals of these satellites. However, the method has not been tested in an environment with strong multipath effects, such as that of a ship. The efficiency of the method for marine applications should be further investigated. For airborne applicarions the efficiency of the SNR-weighting has been verified by several tests with small airplanes.

Acknowledgments

Dr. Michail Gianniou received his degree in Geodesy from the Aristotle University of Thessaloniki, Faculty of Rural and Surveying Engineering, in 1992. He was a student of Prof. L. Mavridis. From 1993 he was a Ph.D. candidate at the Institute of Physical Geodesy, Technical University of Darmstadt, holding a DAAD-scholarship. He received his Ph.D. in Satellite Geodesy in 1996.

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