An Advanced Real-Time Algorithm for Code and Phase DGPS

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ABSTRACT

In the last years the advances in the receiver technology improved substantially the precision of the code pseudoranges. This is important not only for code DGPS but also for carrier phase DGPS, as the code double-difference (DD) solution is a crucial aspect by the most On-The-Fly algorithms. Good code DD coordinates allow a small search space for the ambiguity fixing, which results in fast and reliable ambiguity resolution.

This paper describes a real-time algorithm, that improves the accuracy and precision of the code double-differences. The basic idea of the algorithm has also been applied to phase DGPS. Here, the advantage of the algorithm is not so much the accuracy enhancement, as the efficiency of the algorithm to detect signal distortions and to consider their magnitudes by the adjustment. This becomes particularly important in the case of interferences.

To demonstrate the efficiency of the algorithm, field measurements using geodetic receivers of the latest generation are here analyzed. This data analysis verifies the superiority of the new algorithm, especially in a noisy environment.

1. INTRODUCTION

The GPS market expands continuously and the requirements of the users are growing. The receiver manufacturers response to this request with improved equipment. The geodetic receivers of the latest generation have a significantly improved performance. Dual frequency code data and full wavelengths on L2, even under Anti-Spoofing (AS), are standard characteristics of them. Besides these features, modern receivers supply measurements of better quality. Some years ago the noise level of the code pseudoranges was in the order of a few meters. Today, it is some decimeters.

These improvements must be considered by the development of data processing algorithms. A good example to explain this are the code double-differences (DD). Improved acquisition of weak signals means primarily more precise measurements to low satellites. Should we still treat low satellites as we have done ten years ago using dramatically inferior receivers? What is actually the impact of these observations on the positioning precision? These questions will be discussed in this paper.

Close related to this problem is the question of weighting the observations. As a rule, measurements (code or phase) below 15 or 20 degrees are not used and all others are uniformly weighted. This is a rule of thumb, but nowadays a rough one! The physical interpretation of it could be expressed as: "All observations to satellites above the limit are of exactly the same precision and all the rest are so bad that they should not be used at all". Obviously, this strategy is quite unrealistic and should be reconsidered. A method that overcomes this problem is proposed here and validation tests are described.

2. THEORETICAL BACKGROUND

In general, the precision of the GPS coordinates depends on two parameters: the geometry of the used satellites and the precision of the observations. The first is expressed by means of the Dilution Of Precision (DOP) and the second by the standard deviation of the observables. For DGPS these correspond to the Relative DOP (RDOP) (Goad, 1988) and to the standard deviation of the DDs.

2.1 Impact of low satellites

The DOP as well as the redundancy of the system become better when more satellites well distributed in the sky are used. Thus, low satellites near the horizon improve the DOP. But we do not use them, because their signals travel a long path through the atmosphere and are very noisy (big standard deviation). To achieve the optimal results, we make a compromise rejecting satellites below an elevation cut-off angle. We lose in geometry but we gain in observation quality. Obviously, the optimal limit depends strongly on the performance of the receiver when tracking satellites at low elevations.

It is well known that the precision of the estimated parameters in a least squares adjustment can be determined without having the observations. All we need is the design matrix and the standard deviation of the measurements. Thus, having certain precision requirements for the coordinates we could theoretically compute the corresponding elevation limit. Unfortunately, in practice this is not reliable. A first reason for that is the imperfect modelling of the observations. By building DDs the most undesired effects cancel out but

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Presented at the DSNS'96 Conference, May 20-24, 1996, St. Petersburg

not completely. The analytical estimation of the remaining error is very difficult. A second reason is that the standard deviation of the observations depends on the elevation in a quite complicated way. So, the best way to find the impact of low satellites is to evaluate field data.

Many data tests with geodetic receivers showed that low satellites improve the code DD solution. The situation for the phases is much more complicated and must be further investigated.

2.2 Usual weighting

As already mentioned, the DDs are usually uniformly weighted. Assuming σ to be the standard deviation of the (undifferenced) pseudorange and considering uncorrelated observations the weight matrix of the DDs is (Hofmann-Wellenhof et. al., 1994):

$$P = \frac{1}{2\sigma^2}\frac{1}{n+1}\begin{bmatrix} n & -1 & -1 & \cdots \\ -1 & n & -1 & \cdots \\ -1 & -1 & \ddots & \\ \vdots & \cdots & & n \end{bmatrix} \tag{1}$$

where n is the number of DDs.

In this consideration there are two questionable points:

- We apply the error propagation considering zero correlation between the undifferenced observations. But on the other hand we build double-differences to make the common effects (non-zero correlation) cancel out.
- We assume that all observations are of the same quality (common standard deviation).

2.3 Advanced weighting

These two points are actually very close related. Although - strictly speaking - it is not correct to use equation (1) we do this because of the difficulty to compute the standard deviation of every observation and the covariance between them. We cannot start by estimating the noise of the undifferenced observation in the conventional way and the applying error propagation. For example, the precision of a code pseudorange is very poor, let us say 30 m one sigma. The main reason for that is Selective-Availability (SA). For simplicity let us also assume that this it is the only reason. Applying error propagation assuming stochastic independence, the one sigma for the code-DD would be 60 m, which is a very bad estimation. For modern receivers the noise level of the code-DDs is about 0.2-1.5 m. Of course, the mistake was to ignore the correlation that originates from the SA. The computation of the covariances between the four pseudoranges that build a DD is an extremely difficult task.

There are two good ways to overcome this difficulty:

- to estimate directly the noise of the DDs or
- to estimate the noise contribution of only that effects that do not vanish in the DDs. Then, error propagation for independent observations can be applied.

The first way is easier but there is the problem of estimating the covariance, which is equal to the variance of the single-difference (SD) to the reference satellite. As many effects remain in the SDs, the non-diagonal elements of the weight matrix might be greater than the diagonal, a fact that would lead to a negative definite matrix. The second way is more difficult but has the advantage that the error propagation can be applied. This guarantees a positive definite matrix.

A technique for computing directly the noise of the DDs can be found in (Gianniou, 1995). When data sampled at a high rate are numerically differentiated in time, the noise level is recovered. This method can be applied to raw-pseudoranges (code or phase), SDs and DDs. Biases (ambiguities, clock offset etc.) vanish when differentiating in time. The high sampling rate ensures furthermore that long period effects cancel out. This technique works good but requires a pre-processing and thus is not adequate for real-time applications.

Here, the second way will be followed. For estimating the noise level of only that effects that remain in the DDs, the Signal-to-Noise Ratio (SNR) will be used. This quantity gives the strength of the desired signal relative to that of the noise. Generally, it is defined as (Horowitz and Hill, 1980):

$$SNR = 10 \cdot \log_{10}\left(\frac{V_s^2}{V_n^2}\right) dB \tag{2}$$

where V is the RMS value of the voltage and the indices s and n stand for signal and noise, respectively. The bigger the SNR the more precise is the measurement. The most undesired effects that cancel out in the DDs do practically not affect the SNR.

Very close related to the SNR is the carrier to noise power c/n_o. The relation between this quantity and the variance for the code delay lock loop (DLL) is (Ward, 1994):

$$\sigma_{DLL} = \left[\frac{F \cdot B}{c/n_0}\left(1 + \frac{2}{T \cdot c/n_0}\right)\right]^{\frac{1}{2}} \quad chips \tag{3}$$

where F is the DLL discriminator correlator factor, B the code loop noise bandwidth and T the predetection integration interval. The standard deviation for the phase lock loop (PLL) is:

$$\sigma_{PLL} = \left[\frac{B}{c/n_0} \left(1 + \frac{1}{2 \cdot T \cdot c/n_0} \right) \right]^{\frac{1}{2}} \quad rad \tag{4}$$

Obviously, the c/n_o describes the noise of both the code and the phase measurement. However, we do not use these formulas in our method. The first reason is that the involved parameters are usually not given by the manufacturers. The second reason is that the SNR units are not common for all receivers. For example, the TRIMBLE manufacturer gives for the SSE: „The SNR units are not easily specified". In order to use the SNR for the variance computation, we evaluated a formula for the units of SSE. A double differentiation in time has been applied to code and phase DDs to estimate the standard deviations of the code and the phase raw-measurements. Then, two exponential functions:

$$\sigma(SNR) = a + b \cdot \mathrm{e}^{-c \cdot SNR} \tag{5}$$

have been estimated. The evaluation of the parameters a, b, c has not been done in sense of a least squares fitting, because our method described below is not sensitive to small variations of these parameters. Figures 1 and 2 show the estimated standard deviations and the evaluated empirical functions for the code and the phase, respectively. The worth of these functions is that they express quite good the relative variance between observations with different SNR values. The absolute values seem also to be good estimations but this point must be further investigated.

The SA is a good example to demonstrate why the computed variance does not concern effects that vanish in the DDs, because SA does not affect the SNR. In the contrary, interferences for instance, that do not cancel out in the DDs, reduce the SNR. Strictly speaking, there are some influences that affect the SNR and vanish in the DDs. For instance, the atmospheric refraction, which is mostly eliminated in differential mode, reduces the SNR due to signal attenuation. But this concerns both stations and so does not introduce a differential error. An effect that could have negative impact on the noise evaluation by means of SNR is multipath, which cause variations of the SNR. This case must be further investigated.

On hand of the exponential functions (figures 1,2) for the code and phase pseudorange we build the covariance matrix for the DDs applying error propagation. The non-diagonal elements are equal but the diagonal are not. In the following the use of its inverse matrix as the weight matrix will be referred to as advanced weighting.

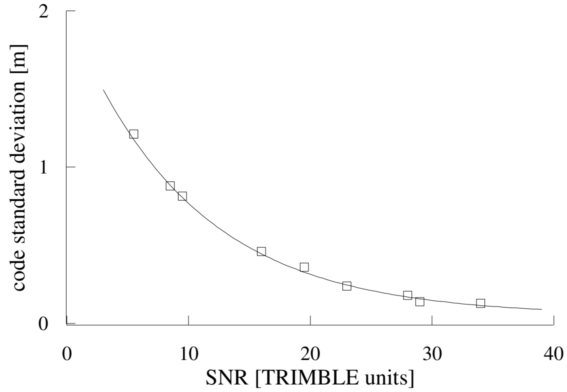

Figure 1: Estimated standard deviations and empirical function (solid line) for code.

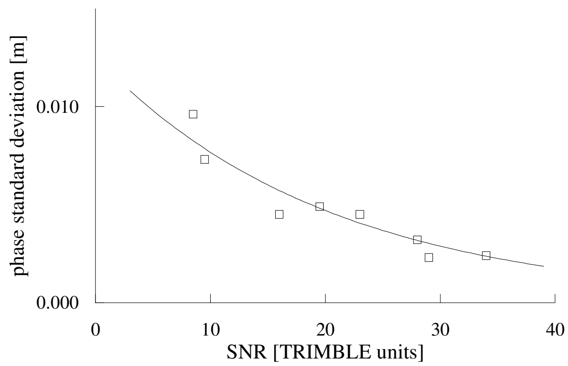

Figure 2: Estimated standard deviations and empirical function (solid line) for phase.

3. DATA ANALYSIS

The impact of low satellites and the potential of the advanced weighting do not depend on the measurement mode: They concern both static and kinematic measurements. For the rapid-static applications they allow the reduction of the occupation time without loss of accuracy (Gianniou, 1995). Here, kinematic data will be analyzed.

3.1 Impact of low satellites

In order to estimate the quality of the code DD coordinates, we use as reference the solution of our OTF algorithm. The difference between the OTF and the code DD coordinates - in the following referred to as error in X, Y or Z - describes both the accuracy and the precision of the code solution.

Analysis of many data sets has shown that satellites below 20° or 15° improve the code DD solution. Here, two examples will be given.

The first data set is from a test with a van in February 1994. Two TRIMBLE 4000SSE receivers have been used together with geodetic antennas with ground-planes. Figure 3 shows the error in Z-coordinate when using all available satellites and only that above 20°. It is interesting that there were many satellites (seven) above 20°. The difference between the two cases are two satellites with elevations between 15° and 20°. The use of all satellites improves all three coordinates as can be seen in figure 5.

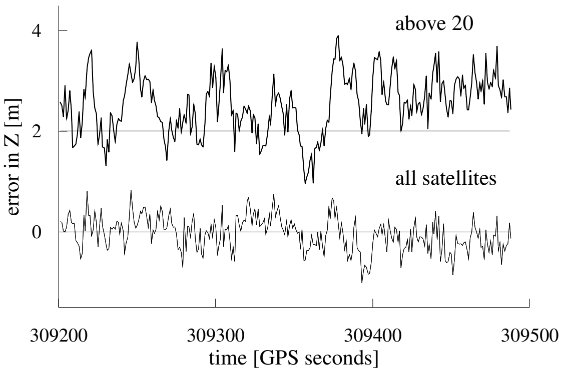

Figure 3: Error in Z-coordinate using all satellites and that above 20° (curve shifted by 2 m).

The maximum distance between reference and rover receiver was 1.5 km. To investigate the influence of this distance, we processed the data using also a far (7.5 km) reference station. Of course, the near one yielded better results, but the improvement when using all satellites was clear in both cases.

The second example is from a flight test with a Dornier DO 228 aircraft in May 1994. Again TRIMBLE SSE receivers and geodetic antennas with groundplanes have been used. Thanks to a special construction, the antenna was mounted in a way that the groundplane lay on the level of the aircraft body. The distance between reference and airborne receiver was up to 1.5 km. Figure 4 shows the error in X-coordinate when using all satellites and when leaving out four satellites below 20°. Their elevations lay between 10° and 17°. The shown data correspond to engine warming up, rolling and taking-off. The improvement of all three coordinates can be seen in figure 5, which gives the standard deviation of the coordinate errors.

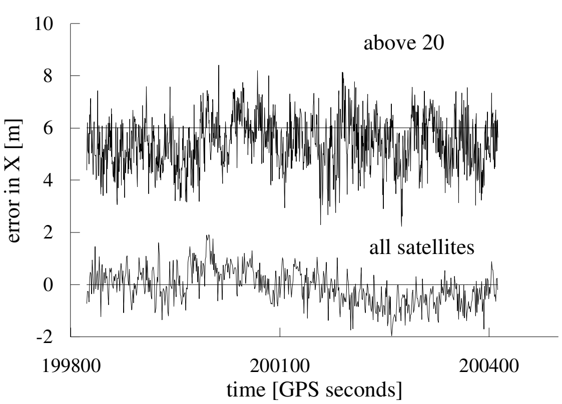

Figure 4: Error in X-coordinate using all satellites and that above 20° (curve shifted by 6 m).

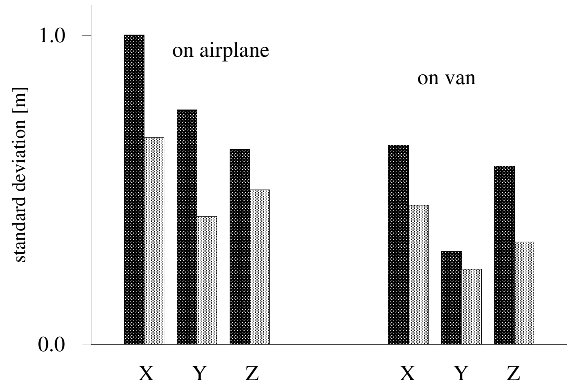

Figure 5: Standard deviation of coordinate errors using all satellites (light bars) and that above 20° (dark bars).

3.2 Advanced versus usual code weighting

In order to demonstrate the efficiency of the advanced code weighting, two examples of kinematic measurements will be given here.

The first is from the flight test described in section 3.1. The elevation cut-off angle for the observations was 10°. As we showed in figure 4 the best results are obtained when all satellites are used in the code DD adjustment. In order to demonstrate the full potential of the proposed method, we will compare the usual and advanced solution when using all satellites. Six to eight satellites were observed, four of them having elevations between 10° and 17°. The error in Z (OTF - code DD coordinate) for the two cases is shown in figure 6.

The second example is from a test measurement with a van on October 25, 1995. On this day AS was disabled and the two TRIMBLE SSi receivers measured P-code pseudoranges. A third SSi, located 7 km away, has been used as an alternative reference station. This receiver has been forced to observe in E-mode, ignoring the P-code. (The E-mode is used by the SSi when AS is enabled, in order to get code and full wavelength phase data on L2). The reason for that was to test our weighting method when the SNR values of one receiver originate from P-code and that of the other from E-mode. Based on our experience we believe that if the relation between noise and SNR is different in the two cases, this difference should be small. This is also verified from the processing of the kinematic data using both the near reference station (P-code) and the far one (E-mode).

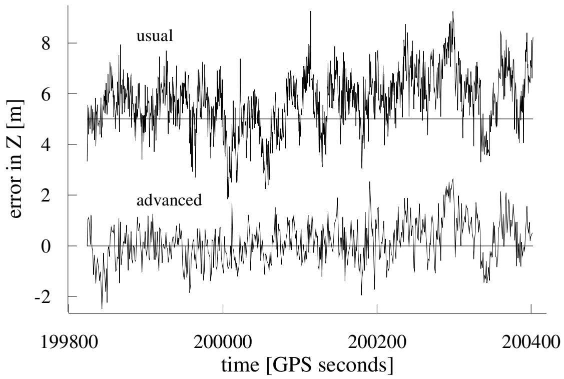

Figure 6: Error in Z-coordinate for advanced and usual code weighting (curve shifted by 5 m).

The error in X for the usual and advanced adjustment using the near reference station (0.2 - 1.4 km) is shown in figure 7. Clearly, the proposed weighting yields much smoother results. Furthermore, the coordinates are closer to the true (OTF) values. Subtracting the minimum from the maximum error, we obtain the width of the interval around the true value in which the coordinates of all epochs lay. These intervals for the tests with the van (using both reference stations) and the aircraft are shown in figure 8.

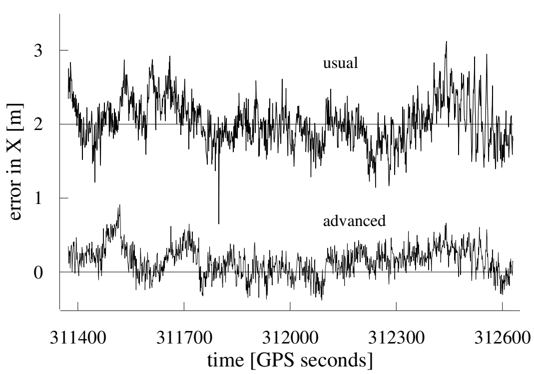

Figure 7: Error in X-coordinate for advanced and usual code weighting (curve shifted by 2 m).

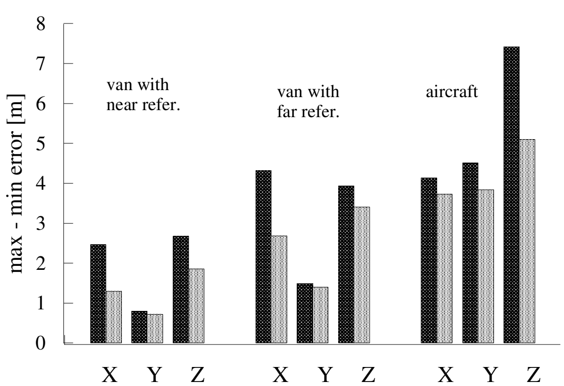

Figure 8: Maximum - minimum error for usual (dark bars) and advanced (light bars) weighting.

3.3 Advanced versus usual phase weighting

The motivation for the development of our method was the big difference in the noise level of the code pseudoranges. In some cases a noisy code DD have a variance about 10 times bigger than a good DD. Outstanding examples can be found in (Gianniou, 1995). For the phases this difference is smaller and does not affect so seriously the coordinates. However, if subcentimeter accuracy is desired then the advanced weighting should be applied.

In general, the epoch by epoch phase solution is very precise. Thus, the advanced phase DD adjustment yields only a small improvement of about 10%-20%. But in extreme cases the improvement can be bigger. We can see such a case if we look at the middle interval in figure 9. The usual solution lies in a 24 mm wide zone around the correct value. The zone for the advanced solution is only 15 mm wide. This corresponds to an improvement of 37%. The data in the figure are collected in static mode at a known baseline. Seven satellites have been used, all above 15°. The signal reception was good and no cycle-slips occurred.

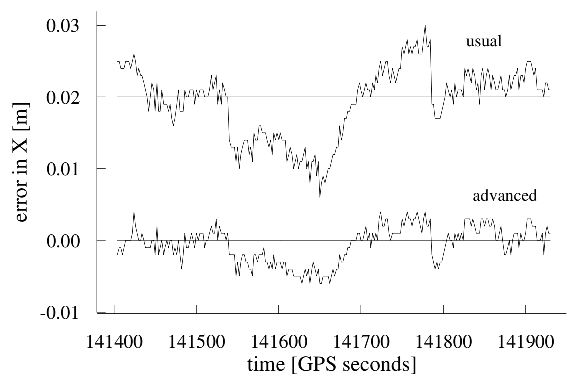

Figure 9: Error in X-coordinate for advanced and usual phase weighting (curve shifted by 0.02 m).

For static measurements, the improvement of the advanced solution is usually as large as in the first and in the last third of the data in figure 7, namely a few millimeters. For kinematic mode we can not make a statement because it is very difficult to have a reference solution with accuracy higher than that of phase DGPS.

4. DISCUSSION

4.1 Impact of low satellites

The impact of low satellites has been investigated by numerous static measurements. It has been found that even very low satellites, e.g. 5°, improve the code DD coordinates. The data were collected with TRIMBLE SSE, SSi and NovAtel L1 cards but we believe that this must be the case by any good geodetic receiver. In addition, there is no reason to believe that the impact of low satellites would be different in kinematic mode.

To verify this, we have analyzed some kinematic measurements. From the data analysis in the previous chapter we can see that the low satellites make the solution more stable. The standard deviations of the coordinate errors are reduced as well as their minimum and maximum values. This can be explained by the increased redundancy of the system in connection with the low noise of the observations near the horizon.

Besides the enhanced precision, there is sometimes a big improvement in the accuracy (absolute value) of the coordinates, as for example in the last 100 seconds in figure 3. In general, this can be explained by the better geometry when using more satellites.

The improvement of the code solution is important not only for code DGPS but also for carrier phase DGPS, especially in real-time. The reason for that is that good code coordinates allow for a smaller searching space when fixing the float ambiguities. An extensive discussion of this point can be found in (Gianniou and Groten, 1996).

4.2 Advanced versus usual code weighting

Besides the positive impact of low satellites, the code solution can be improved using the proposed advanced weighting.

First of all, the advanced adjustment leads to smoother coordinates and consequently to smoother trajectory. In addition, the estimated position is closer to the correct one as can be seen in figures 6 - 8. In the last figure we can see that the biggest improvement is obtained for the most noisy coordinates and amounts up to 48%. Another interesting point concerns the negative peak in the usually weighted data of figure 7. Such peaks appear often when the receiver begins to track a new satellite. At the very first epochs this one measurement can be so bad to cause a considerable error even in overdetermined systems. Such measurements have - at least by TRIMBLE - a very low SNR indicator. Our method gives to such observations very small weight preserving the precision of the solution.

A last aspect to be discussed is the different level of improvement of the three coordinates. As already mentioned, at most are improved the coordinates that are worst estimated. This is also the case when using low satellites (see figure 8). This fact can be explained by the geometry of the DDs. In the same way we can explain why sometimes one coordinate becomes slightly worse. But this is not a problem because this coordinate is always the best one. For example, for the geographical position of Germany this is the Y coordinate. Furthermore, the degradation is negligible compared to the improvement of the other two coordinates.

4.3 Advanced versus usual phase weighting

The advanced weighting of the phases is needed mainly for highest precision applications. However, for ordinary applications it has two advantages. First, it permits the control of the measurements by means of the SNR and so prevents from using quite bad observations that can distort considerably the results. Second, at some certain epochs the improvement can amount more than 1 cm, as for example about the epoch 141660 in figure 9. So, the proposed method becomes effective when a position is determined from few observations. This is always the case in kinematic DGPS.

5. CONCLUSIONS

In this paper the role of low satellites in DGPS has been investigated. The test measurements with the geodetic receivers showed that satellites between 5° and 20° above the horizon improve the code DD coordinates. This is important not only for code DGPS but also for phase DGPS because a good code solution accelerates the OTF algorithm. This reduction of the computation time is particularly important for real-time implementations.

It would be interesting to find out the impact of low satellites on the code DDs for receivers without phase measurement capability and if a lower elevation cut-off angle must be adopted or no limit at all. In addition, the situation for the phase DDs must be further investigated.

A new method for weighting the code and phase DDs has been described and its efficiency has been demonstrated by analysis of field data. The method takes into account the different noise level of the observations used in an epochwise adjustment. The noise level depends primarily on the receiver performance, the elevation of the satellites and environmental effects such as interferences.

The effect of the advanced weighting is similar to that of a filter, i.e. the noise of the estimated parameters is reduced. In comparison with existing filtering algorithms our method has the following advantages. It does not need time to become effective, as for example the phase-smoothing of codes. It is not as time-consuming as the implementation of sophisticated filters. All what is needed is a slight modification of the routine for the computation of the weight matrix. These two aspects make our method particularly adequate for real-time applications. Furthermore, for the advanced adjustment of code DDs there is no need to have expensive equipment with phase measurement capability.

The main reason for increased noise in the observations is the low elevation of the satellite. Some researchers have determined formulas describing the relationship between noise and elevation. The disadvantage of these functions is that they are common for every data set and do not consider occasional parameters. Interferences for instance increase the observation noise and decrease the SNR. The proposed method considers this fact and overcomes the problem to a large extend. An aspect to be further investigated is the behaviour of our method in the case of fluctuating SNR due to e.g. strong multipath.

But in any case the dependence of noise on either elevation or SNR is different for every receiver type. In other words the empirical functions (5) for the code and the phase must be estimated for the available receiver type. The functions shown in figures 1 and 2 are good estimations for the SSE and SSi models.

Our approach of individually weighting every observation will lead to a dramatic improvement in the precision of the single-point positioning in case that SA will be turned off. This case is seriously being discussed in the last years.

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