OPERATIONAL EFFICIENCY MAP & FLOW CHARACTERIZATION FOR STEADY-STATE TWO-PHASE FLOWS IN POROUS MEDIA

Marios S. Valavanides
TEI Athens, Ag. Spyridonos, Athens, GR-12210, Greece, e-mail: marval@teiath.gr

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ABSTRACT
An extended retrospective examination of relative permeability diagrams revealed a latent characteristic of steady-state two-phase flows in porous media, the existence of *optimum operating conditions*, i.e. conditions whereby process efficiency -considered in terms of oil produced per kW dissipated in pumps- attains locally maximum values. A total number of 83 published relative permeability diagrams, pertaining to a variety of flow conditions and core types, were reconstructed into energy utilization diagrams, providing ample evidence on the existence of optimum operating conditions and showing a universal trend that can be cast into an *operational efficiency map* over the domain of the independent variables, the capillary number, Ca, and the oil/water flow ratio, r. This universal map demarcates the overall efficiency of steady-state two-phase flow processes in terms of pertinent system parameters. The map not only provides a guiding tool for designing more efficient processes, but can be used for the *normative characterization* of two-phase flows as to the predominance of *capillary* or *viscous* effects. The proposed concept is based on the existence of a unique *locus of optimum operating conditions*, r*(Ca), for each particular oil-water-p.m. system.

INTRODUCTION
Conventional relative permeability diagrams are records of experimental evidence on the phenomenology of steady-state two-phase flow in porous media processes. They contain untapped latent information that may enrich our knowledge and deeper understanding of the process and may be used constructively to back-up, or provide further evidence on theoretical inferences. A particular inference of utmost importance is the existence of conditions whereby the efficiency of steady-state two-phase flow processes -considered in terms of oil produced per kW dissipated in pumps- attains locally maximum values.
Optimum operating conditions (*OOC*) were first predicted and identified by the *DeProF* theory for steady-state two-phase flow in pore networks [1]. The operational efficiency of the process can be measured by the energy utilization index,

$$f_{EU} = r/W \tag{1}$$

where: r is the oil/water flowrate ratio and $W \equiv \widetilde{W}\widetilde{k}\widetilde{\mu}_w(\widetilde{\gamma}_{ow}Ca)^{-2}$ is the reduced mechanical power dissipation (including the effect of bulk viscosities and interfacial hysteresis on strain rates), whereby $\tilde{k}, \tilde{\mu}_w, \tilde{\gamma}_{ow}$ are the absolute permeability, the water viscosity and the o/w interfacial tension respectively and Ca is the capillary number.

RE-EXAMINATION OF RELATIVE PERMEABILITY DIAGRAMS

Data sets and diagrams of measured relative permeabilities, $k_{ri}(S_w)$, i=o,w, from 19 published studies, comprising 83 laboratory runs of immiscible steady-state flows in various types of real cores as well as glass and virtual pore network models, have been transformed into corresponding energy utilization data sets, $f_{EU}(r)$, to check if any latent information –such as the existence of optimum operating conditions- can be revealed. The results of this re-examination are recorded in a technical report [2].

Transformation of relative permeability data into energy utilization data

The transformation originally introduced in [1],

$$r = \frac{\tilde{q}_o}{\tilde{q}_w} = \frac{k_{ro}/\tilde{\mu}_o}{k_{rw}/\tilde{\mu}_w} = \frac{1}{\kappa}\frac{k_{ro}}{k_{rw}} \quad \text{and} \quad f_{EU} = \frac{r}{W} = \frac{k_{ro}}{\kappa(r+1)} = k_{rw}\frac{r}{r+1} \qquad \text{(2a,b)}$$

with $\kappa = \tilde{\mu}_o/\tilde{\mu}_w$ the viscosity ratio, is valid for steady-state two-phase flows in p.m. It was implemented in reconstructing measured relative permeability vs saturation data sets, $\{k_{ro}, k_{rw}, S_w\}_j$, into corresponding energy utilization vs flowrate ratio data sets, $\{f_{EU}, r\}_j$.
The flow ratio, r, and water saturation, S_w, are inversely related and, in the limits,

$$\text{as } S_w \to 1^- \ \Rightarrow \ r \to 0^+ \quad \text{and} \quad \text{as } S_w \to 0^+ \ \Rightarrow \ r \to +\infty \qquad (3)$$

Note that the independent variables of the steady-state 2-phase flow in p.m. are Ca & r but not S_W. Considering saturation as an independent variable is just conventional and inefficient. Relative permeabilities plotted on Ca×r describe more efficiently the process.
A typical reconstruction of $\{k_{ro}, k_{rw}, S_w\}_j$ into $\{f_{EU}, r\}_j$ data sets is depicted in Figure 1: two sets of steady-state relative permeability diagrams (sources [3] & [4]), are transformed by use of eqns (2) into energy utilization diagrams. Similar reconstructions were produced for a total of 83 steady-state relative permeability diagrams [2], whereby a locally maximum value of energy utilization was observed. A handy spreadsheet for transforming any measured $\{k_{ro}, k_{rw}, S_w\}_j$ data sets into corresponding $\{f_{EU}, r\}_j$ data sets is available online at http://users.teiath.gr/marval/ArchIII/relpermtrans.xls.
Any set of relative permeability diagrams comprise two concave curves that intersect to a fixed [refer to eqn (2a)] value of r_x:

$$k_{ro}(r_x) = k_{rw}(r_x) \quad \Rightarrow \quad r_x = 1/\kappa \qquad (4)$$

A meticulous (and lengthy) analysis, considering the physical characteristics of the process at the far end of the Ca spectrum, Ca→+∞, yields the asymptotic value of the flow ratio, r_∞^*, that needs to be reached to operate the process at optimum efficiency

$$r_\infty^* = \lim_{Ca\to\infty} r^* = k_{ro}(r_\infty^*)/\left[\kappa k_{rw}(r_\infty^*)\right] \ \Rightarrow \ r_\infty^* = \lim_{Ca\to\infty} r^* = 1/\sqrt{\kappa} \qquad (5)$$

The corresponding (upper) limit value of the operational efficiency, $f_{EU\infty}^*$, is

$$f_{EU\infty}^* = 1/\left(1+\sqrt{\kappa}\right)^2 \qquad (6)$$

Universal trends observed in steady-state two-phase flows in porous media

The 83 relative permeability data, $\{k_{ri}\}_j$, i=o,w and the respective $\{f_{EU}\}_j$ vs $\{\log r\}_j$ values, have been plotted in uniform diagrams, incorporating the fixed/asymptotic values $r_x, r^*_\infty, f^*_{EU\infty}$, to demarcate the limits of the efficiency of the respective flows [2]. For each oil-water-p.m. system examined -two typical cases are depicted in Figure 1, the following universal characteristics are observed:

1) Relative permeabilitiy values plotted against $\log r$ under fixed Ca, outline S-shaped curves. This is useful when trying to interpolate sparse relative peremeability data.

2) There *always* exists an optimum flow rate, r*, at a certain Ca value, for which the process operational efficiency becomes (locally) maximum. The overall trend of $f_{EU}(Ca,r)$ against the viscosity ratio, κ, is similar to the results of the *DeProF* simulations [1].

3) The flow ratio for optimum operation, r*, is located at a distance, $d = \log\left[r^*(Ca)\sqrt{\kappa}\right]$, from the asymptote of the maximum efficiency flow ratio, $r^*_\infty = 1/\sqrt{\kappa}$.

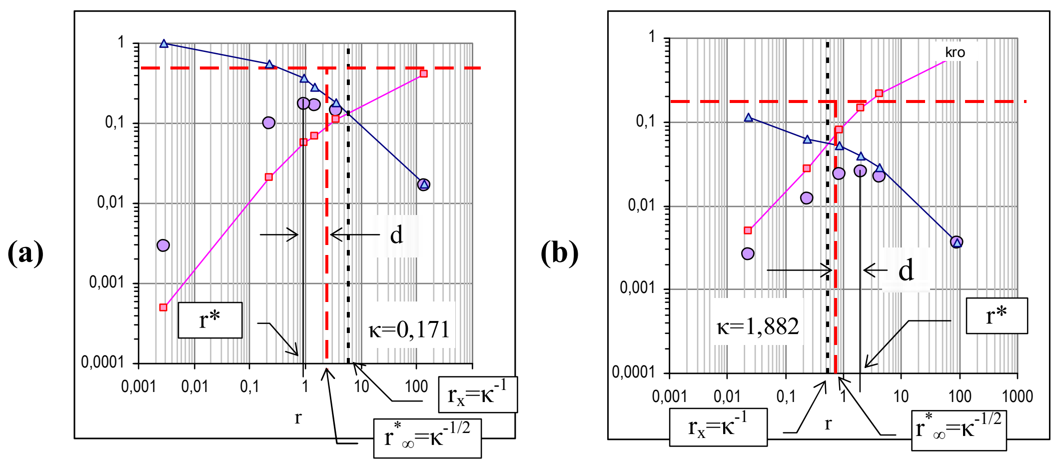

Figure 1: Data sets of relative permeability for 'oil' (□) & 'water' (△) and energy utilization index, f_{EU}, (○) against o/w flow ratio, r, pertaining to (a) favorable and (b) unfavorable viscosity ratios, κ, and different cores. The vertical dotted line (in black) indicates the critical flowrate, r_x, for which the relative permeabilities of oil & water are equal. The dashed (horizontal & vertical) lines (in red) represent the asymptotic values of the flow ratio for optimum operational efficiency, r^*_∞, and the energy utilization index, $f_{EU}(r^*_\infty)$, in pure viscous flow conditions (Ca→∞).

OPERATIONAL EFFICIENCY MAP OF STEADY-STATE FLOWS

The analysis expressed through eqns (2-6), the observations on the reconstructed relative permeability diagrams [2], and the *DeProF* model predictions [1], have been combined together to cast the universal *operational efficiency map* for steady-state two-phase flows in porous media. An impression of the map pertaining to favorable viscosity ratio, $0 < \kappa < 1$, is depicted in Figure 2.

The intersecting curves residing on the Ca=Ca$_3$ plane, represent the relative permeabilities for oil (dashed) and water (dash-dotted) as a function of the flow ratio, r, under fixed capillary number, say Ca$_3$. The relative permeability curves intersect at a point projecting on $(\mathrm{Ca}_3, 1/\kappa)$ in the Ca×r plane. The intersection points from all pairs of relative permeability curves project on the straight dotted line, $\mathrm{r} = 1/\kappa$.

The hook-shaped thick solid curve represents the intersection of the surface of the energy utilization index, f_{EU}(Ca,r), with plane Ca=Ca_3. Similar hook-shaped curves are profiling the f_{EU}(Ca,r) surface as it virtually intersects with arbitrary planes Ca=Ca_1 & Ca=Ca_2.

The thick curve, $f_{EU}(r^*)$, delineates the ridge of the energy utilization surface f_{EU}(Ca,r) and projects on the *locus of optimum operating conditions*, r*(Ca), for which maximum process operational efficiency is attained (the *OOC locus*). The asymptotes of r*(Ca) and $f_{EU}(r^*)$ as Ca→+∞ are depicted with dashed lines.

At any fixed-Ca plane, the flow ratio for optimum operation, r*(Ca), is located at a distance, $d = \log\left[r^*(\mathrm{Ca})\sqrt{\kappa}\right]$, from the asymptote of the maximum efficiency flow ratio, $r^*_\infty = 1/\sqrt{\kappa}$. The shift d is large when capillarity effects are dominant and diminishes to zero at very high flow rates ($\mathrm{Ca} \to \infty$) whereby bulk viscosity predominates the flow.

The energy utilization index along the OOC locus, f_{EU}[r*(Ca)], is S-shaped as it is framed between 0 and $f_{EU}\left(r^*_\infty\right) = 1/\left(1+\sqrt{\kappa}\right)^2$.

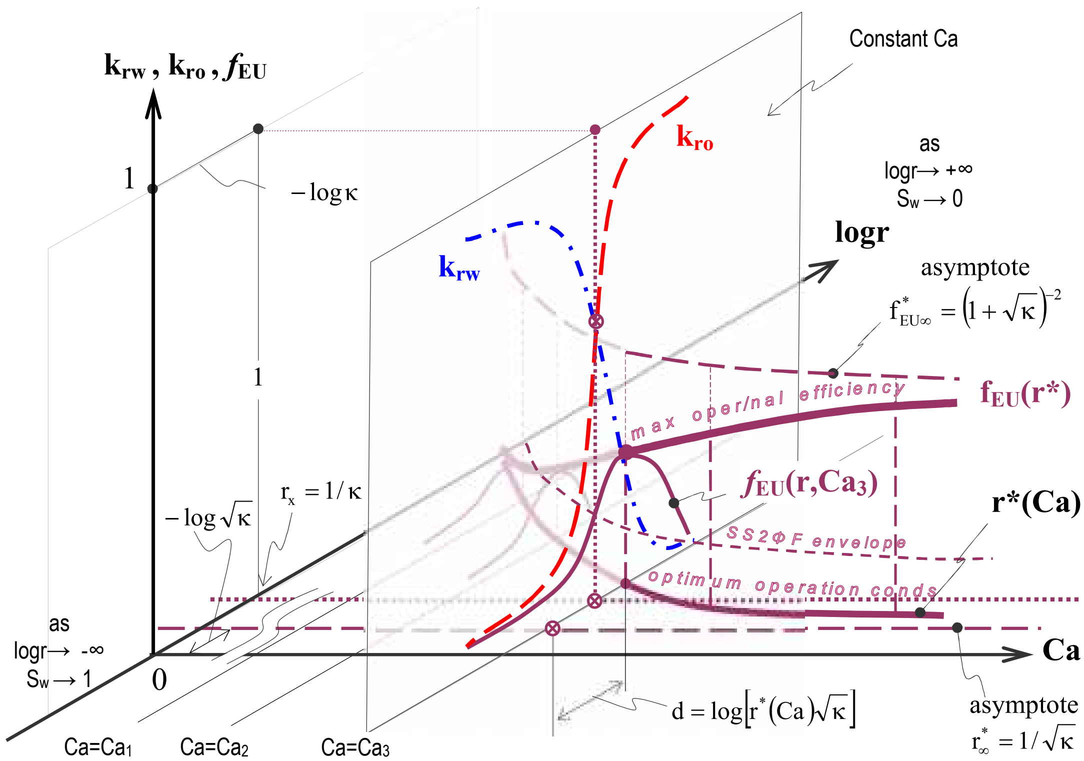

Figure 2: Universal, operational efficiency map describing steady-state two-phase flow in porous media (impression for favorable viscosity ratio, 0<κ<1).

The map demarcates the operational efficiency of any oil-water-p.m. system and provides pertinent scaling laws for critical macroscopic variables.

The process marginal efficiency can be measured by an index, b, defined as

$$b(Ca, r) = f_{EU}(Ca, r)/f^*_{EU\infty} = f_{EU}(Ca, r)\left(1+\sqrt{\kappa}\right)^2 \quad (7)$$

Clearly. $0 < b < 1$ and this may be used to compare the efficiencies of processes for systems with different viscosity ratios, κ.

CHARACTERIZATION OF TWO-PHASE FLOWS IN P.M.

Capillarity and interfacial phenomena account for the nonlinearities observed during two-phase flows in p.m. even in steady-state conditions. Capillarity predominates the flow in the lower end of the flow variables regime, i.e. for "relatively low" Ca values. In contrary, at the higher end of the flow variables regime, for "relatively high" Ca values, the flow is dominated mainly by the bulk viscosities of the two phases.

A new methodology is proposed for the *normative characterization* of two-phase flows in porous media as *capillary* or *viscous*. Conceptually, it is based on the observation that the OOC locus, r*(Ca), shows a significant mutation as Ca is increased -from "small" to "large" values. For <u>any</u> oil-water-p.m. system this mutation is expressed by the particular form of <u>the</u> OOC locus, r*(Ca), for <u>that</u> system. Referring to Figure 2, there are two zones of equal width, $\log\sqrt{\kappa}$. The part of the OOC locus residing outside the $|\log\kappa|$-wide zone is sensitive to Ca, and capillarity dominates the flow. The other part of the locus, residing within the $|\log\kappa|$-wide zone, gradually becomes insensitive to Ca (as Ca increases).

In general, the locus r*(Ca) may be approximated by a scaling function of the form

$$\log[r^*(Ca)] = -\log\sqrt{\kappa} + \frac{A}{Ca^{\,B}}, \quad A, B > 0 \quad (8)$$

where parameters A & B are positive real numbers that can be estimated from a set of laboratory determined optima $\{r^*_i, Ca_i\}$ for any particular oil-water-p.m. system.

Considering the cases for pure favorable or unfavorable viscosity ratios, i.e. $\kappa \neq 1$, and the functional form of the OOC locus [eqn. (8)], a critical value of the capillary number, Ca_{cv}, segregating capillary- from viscosity- dominated flows may be determined as,

$$Ca_{cv} = \begin{Bmatrix} \left(-2A/\log\kappa\right)^{\frac{1}{B}} & 0<\kappa<1 \\ \left(2A/\log\kappa\right)^{\frac{1}{B}} & 1<\kappa \end{Bmatrix} = \left(\frac{2A}{|\log\kappa|}\right)^{\frac{1}{B}}, \quad \kappa \neq 1 \quad (9)$$

For cases where $\kappa = 1$, considering the continuous dependence of the locus r*(Ca) on κ, as κ crosses 1, the fixed value $r_x = 1/\kappa$ coincides with the asymptote $r^*_\infty = 1/\sqrt{\kappa}$, therefore $r_x = r^* = 1$, and OOCs are met for <u>fixed</u> r=1, whereby the viscosity contribution in each phase is balanced for any Ca and, consequently, W is minimized. The physical interpretation is that the viscosities of the two phases are equal and the power dissipated within the process is indifferent to saturation and it is only regulated by interfacial friction losses. The locally maximum value attained by f_{EU}, depends on the total mechanical power dissipation that, in turn, *depends on the structure* of the porous medium i.e. not only on absolute permeability, but also on tortuosity, pore size distribution and pore size correlations, micro-roughness, fractal characteristics etc. For $\kappa = 1$, eqn (8) gives A=0 and the value of B is regulated by the capillarity effects that are

induced by the p.m. microstructure. Therefore, to estimate the value of B from laboratory measurements, the functional form (and values) of f_{EU} should also be taken into account.
In addition to flow characterization, the one-to-one correspondence between the form of the OOC locus and the oil-water-p.m. system, expressed by eqn (8), can also be used *to characterize the structure* of any particular p.m. with a triplet $\{\kappa, A, B\}$, meaning that parameters A & B, record the effect of the p.m. structure on a variety of flows with specifically selected pair(s) of fluids, κ.

CONCLUSIONS

A re-examination of published relative permeability diagrams provided experimental evidence on the existence of optimum operating conditions in steady-state two-phase flow in p.m. processes, as predicted by the *DeProF* theory. A total of 83 published relative permeability diagrams for a variety of flow conditions and core types, were reconstructed into energy utilization diagrams. The latter show a universal trend that can be cast into an *operational efficiency map* over the domain of the independent variables, the capillary number and the oil/water flow ratio. This universal map demarcates the overall efficiency of steady-state two-phase flow processes in terms of pertinent system parameters and could provide potentially large marginal benefits in industrial applications. In addition,
A normative methodology for evaluating the viscous/capillary character and efficiency margin of 2-ph flows in p.m. was introduced. The methodology is based on the definition of a critical value for the capillary number that, depending on the values of pertinent system parameters, segregates capillary- from viscosity- dominated flow regimes.
An outstanding problem that merits further elaboration is the development of an efficient protocol for the laboratory measurement of the values of system parameters A & B, appearing in scaling forms similar to eqn (8), for every particular oil-water-p.m. system.

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