Operational Efficiency Map of Steady-State Two-Phase Flow in Porous Media Processes

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1 Introduction and summary

Experimental evidence on the phenomenology of steady-state two-phase flow in porous media is recorded in the well-known relative permeability curves published in the literature. A retrospective examination of such curves revealed a very interesting process characteristic, the existence of optimum operating conditions, i.e. conditions whereby process efficiency -considered in terms of oil produced per kW dissipated in pumps- attains locally maximum values. Optimum operating conditions were first predicted and identified by the *DeProF* theory [1] for steady-state two-phase flow in pore networks. The operational efficiency of the process is measured by the energy utilization index

$$f_{EU} = r/W \tag{1}$$

where: r is the oil/water flowrate ratio and $W \equiv \tilde{W}\tilde{k}\tilde{\mu}_w\left(\tilde{\gamma}_{ow}Ca\right)^{-2}$ is the reduced mechanical power dissipation (including the effect of bulk viscosities and interfacial hysteresis on strain rates). Ca, the capillary number, and r, the oil-water flowrate ratio, are the process operational parameters; $\tilde{W}$ is the specific rate of mechanical energy dissipation of the two phase flow, and $\left(\tilde{\gamma}_{ow}Ca\right)^2/\left(\tilde{k}\tilde{\mu}_w\right)$ the corresponding rate for one-phase flow of water. Extensive simulations using the *DeProF* mechanistic model revealed the existence of optimum operating conditions in the form of a smooth and continuous locus, [r^*(Ca)] in the domain of the process operational parameters, see diagrams in Fig. 1.

In the present work, an operational efficiency map for state–state two-phase flow in porous media processes is furnished (Figure 3). This map demarcates the operational efficiency of such processes. It is the result of analysis based on physical considerations and provides a guiding tool for designing more efficient processes. A simple transformation of published $k_{ri}(S_w)$, i=o,w relative permeability diagrams pertaining to steady-state two-phase flows in real porous media (sand packs) into corresponding energy utilization diagrams, $f_{EU}(r)$, was implemented. By plotting the relperm data, k_{ri}, and the respective, f_{EU}, values versus logr, certain interesting observations are made.

(a) Relative permeabilities attain the form of an S-curve when expressed as a function of logr. This can be useful to interpolate sparse relperm data.
(b) The flowrate ratio values for which the flow attains its locally maximum efficiency, r*, are always shifted to higher values than that (value) for which the relative permeabilities of oil & water are equal, r_x. The shift can be used as a norm for evaluating the capillarity characteristics of the flow.

These observations show a universal trend as is indicated by the operational efficiency map. This map resolves, in a consistent and rational manner, to what extent- disconnected oil flow and associated capillarity effects regulate the flow. The map is validated by direct comparison against *DeProF* theory predictions, Fig. 1, and published relative permability measurements pertaining to steady-state two-phase flows in pore networks and real porous media. It is similar to the map furnished by Lenormand and coworkers ([2], [3]) demarcating the two-phase flow patterns observed for various domains of the values of the capillary number and the viscosity ratio. This new map provides not only qualitative information but also valuable quantitative information with respect to operational efficiency aspects of the sought process.

2 The transformation of relperm curves to energy utilization curves

The transformation originally introduced in [1],

$$r = \frac{\tilde{q}_o}{\tilde{q}_w} = \frac{\tilde{U}_o}{\tilde{U}_w} = \frac{k_{ro}/\tilde{\mu}_o}{k_{rw}/\tilde{\mu}_w} = \frac{1}{\kappa}\frac{k_{ro}}{k_{rw}} \quad \text{and} \quad f_{EU} = \frac{r}{W} = \frac{k_{ro}}{\kappa(r+1)} = k_{rw}\frac{r}{r+1} = k_{ro}\left(\frac{k_{ro}}{k_{rw}} + \kappa\right)^{-1} \qquad (2)$$

where $\kappa = \tilde{\mu}_o/\tilde{\mu}_w$ is the viscosity ratio, is valid for steady-state two-phase flow in porous media and helps us reconstruct the -experimentally derived- relperm vs saturation curves (k_{ro}, k_{rw} vs S_w) into energy utilization vs flowrate ratio curves, f_{EU} vs r.

An indicative reconstruction is depicted in Figure 2; two sets of relperm curves furnished by Bentsen [4], are transformed through eqs (2) into energy utilization diagrams. Similar reconstructions were produced for available sets of steady-state relperm diagrams and show a universal trend.

3 Demarcation of the operational efficiency of steady-state 2-ph flow in p.m. processes

Examining the partial derivative of f_{EU} with respect to r from eqs (2b), results in a differential equation for k^*_{ro} and k^*_{rw} corresponding to the local maxima of f_{EU} (at r^*) for fixed Ca,

$$\left.\frac{\partial}{\partial r} f_{EU}\right|_{r=r^*} = 0 \Rightarrow \quad k^*_{ro} = C_o(Ca)(r^*+1) \quad \text{and} \quad \left.\frac{\partial}{\partial r} f_{EU}\right|_{r=r^*} = 0 \Rightarrow \quad k^*_{rw} = C_w(Ca)\frac{r^*+1}{r^*} \qquad (3)$$

whereby the integration constants are functions of Ca and describe the capillary characteristics of the physical system (oil-water-p.m.); these are linearly dependent as $C_o(Ca) = \kappa C_w(Ca)$.

All relperm diagrams show attain a crossing sabre pattern, i.e the the two curves intersect at a certain value of water saturation, say S_{wx}., corresponding –through eq (2a)- to a value r_x., such that,

$$k_{ro}(r_x) = k_{rw}(r_x) \quad \Rightarrow \quad \kappa = 1/r_x \qquad (4)$$

A meticulous analysis considering the physical characteristics of the process in the far end of the Ca spectrum, i.e. when Ca→+∞, yields the asymptotic value of the flowrate ratio, r^*_∞, that needs to be maintained to operate the process at optimum efficiency

$$r^*_\infty = k_{ro}(r^*_\infty)/\left[\kappa k_{rw}(r^*_\infty)\right] \quad \Rightarrow \quad r^*_\infty = 1/\kappa \qquad (5)$$

The corresponding (upper) limit value for the operational efficiency, $f^*_{EU\infty}$, is given by

$$f^*_{EU\infty} = (1+\kappa)^{-1} \qquad (6)$$

Equations (4)-(6) had been incorporated in the diagram of Figure 3 to prescribe the limiting values of the operational efficiency of two-phase flow in porous media processes.

4 Operational efficiency trends observed for steady-state 2-ph flow in p.m. processes

In all relperm diagrams that were retrospectively examined, there exists an optimum operating condition, r*, and the trend in f_{EU}(Ca,r) with respect to the viscosity ratio, κ, is similar to that predicted by *DeProF* simulations (compare Figs. 1 & 2).

In addition, the flowrate ratio for optimum operation, r*, is close to- but not coinciding with- the flowrate value, r_x, corresponding to the crossing of the relperm curves for oil & water.

The "distance" (shift) between conditions r* and r_x, is denoted as d=(logr*+logr_x); this shift is large when capillarity effects are dominant and diminishes to zero at very high flow rates, or, relatively large Ca values, when the role of bulk viscosity is predominant.

The existence of 'optimum conditions' for oil transport in steady-state two-phase flow in pore networks is a consequence of the remarkable internal adaptability of the flow to externally imposed flow conditions (Ca, r) and its inherent characteristic in trading-off between viscosity and capillarity and self-adjusting the total flow between connected and disconnected moving-oil balance.

5 Conclusions

Detecting and setting optimum operating conditions in a real process could eventually increase the process efficiency; that, in turn, could provide potentially large marginal benefits in industrial applications, such as enhanced oil recovery, fuel cells etc. The operational efficiency of such processes is now demarcated –at least to a certain extent; nevertheless, there is still much to be done on the field of *ab-initio* pinpointing of the optimum operating conditions when the process needs to be operated at predetermined values of capillary number or flowrate ratio.

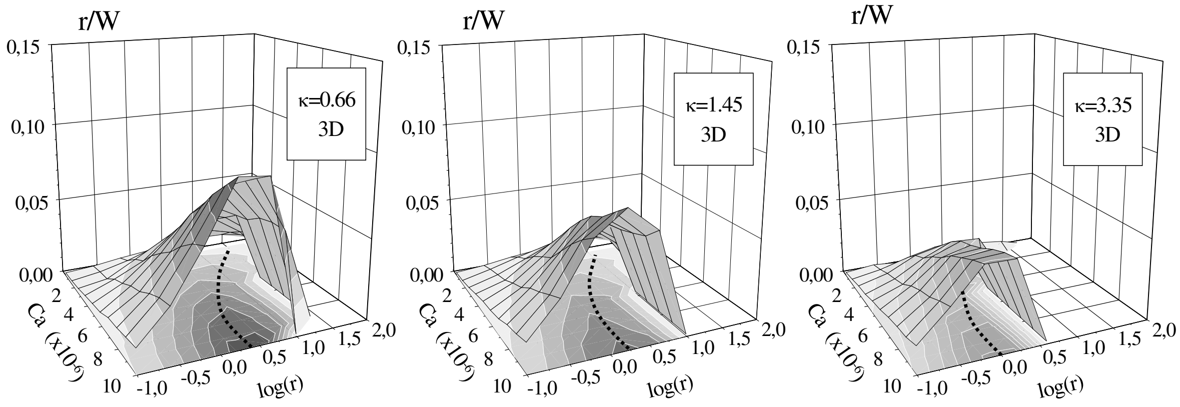
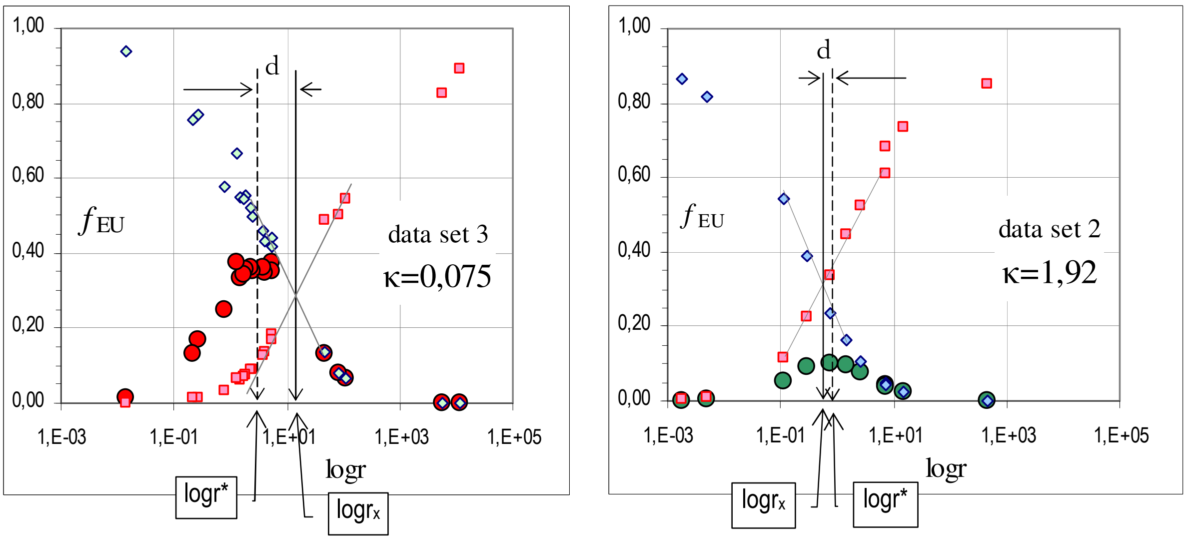


Figure 1: Energy utilization factor, f_{EU}=r/W, as a function of Ca and r. The diagrams pertain to 3D pore network *DeProF* simulations for three o/w systems with viscosity ratios κ = 0,66, 1,45 and 3,35 [1]. The dashed lines represent the projection of the ridge of the f_{EU}(Ca,r) surface on the (Ca,r) plane. These lines define the loci r*(Ca) of the optimum operating conditions for the three systems considered.



(a) **(b)**

Figure 2

Steady-state relative permeability for oil (□) and water (◊) and energy utilization index, f_{EU}, (●) corresponding to flowrate ratio values, r, for two-phase co-current flow in a fine sand pack. The values of r and f_{EU} are computed by transforming - through eqs (2)- the original measurements published by Bentsen [4]. The data sets pertain to (a) favorable and (b) unfavorable viscosity ratios, κ.

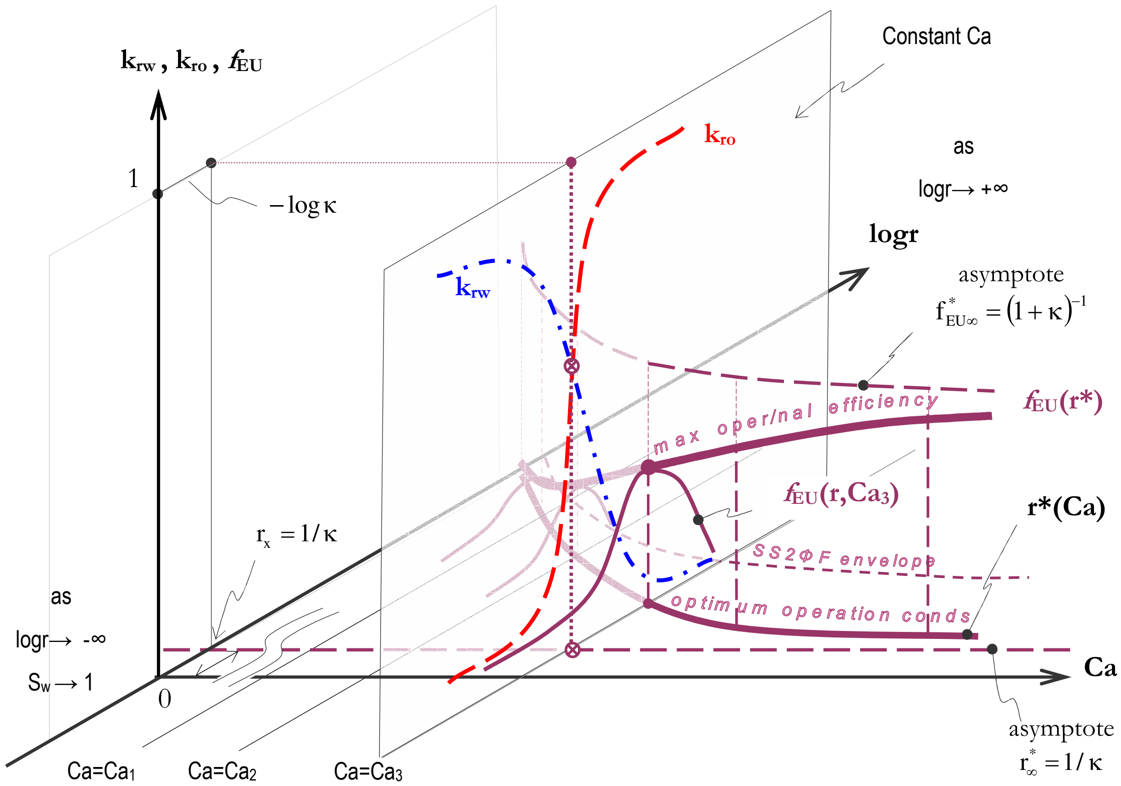


Figure 3

Operational efficiency map of steady-state 2-ph flow in porous media. Thick solid curves provide a schematic representation (phantom view) of the universal form of the energy utilization coefficient, f_{EU}=r/W. The intersections of the f_{EU}(Ca,r) surface (similar to those presented in Fig.1) with planes vertical to the Ca axis, located at arbitrary points (Ca_1, Ca_2) are shown opal behind a plane at Ca_3. The thick violet curve, f_{EU}(r*), delineates the ridge of the energy utilization surface f_{EU}(Ca,r) and corresponds to optimum operation conditions, , r*(Ca), whereby maximum efficiency is attained. The relative permeability curves for oil and water, k_{ro} and k_{rw}, appear on the Ca=Ca_3 plane. The asymptotes of r*(Ca) and f_{EU}(r*) as Ca→+∞, depicted with dashed lines, are given respectively by $r^*_\infty = 1/\kappa$ and $f_{EU}(r^*) = (1+\kappa)^{-1}$.

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