New numerical and computational methods for the solution of differential equations with applications in environmental issues

Working package 4, The project results

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The frame work of WP4

WP 4 was mainly focusing on the application of the new models, techniques and algorithms developed within the framework of WPs 1-3 for the optimization and the local adaptation of the direct outputs of numerical environmental prediction models. Based on a solid mathematical background supported by Information Geometry and Statistical techniques, new versions of Kalman filters have been developed able to optimally combine direct meteorological model forecasts and local available observations in order to provide local adopted, bias free wind and wave predictions reducing at the same time the associated uncertainty of the final forecasts. The new optimization methods have been tested in selected areas of Greece where the use of renewable energy sources is of critical importance and interest for wind speed and wave height predictions.

The procedure followed

The parameter employed for the tests is the wind speed at 10 m as simulated by the atmospheric prediction system Skiron. This choice has been made mainly for two reasons:

- Wind speed is one of the most critical parameters of numerical weather forecasting having direct impacts in renewable energy site assessment, maintenance and forecasting.
- On the other hand, due to its variable and discontinuous nature and the increased correlation with local area characteristics, it is one of the most commonly biased parameters by meteorological forecasting models.

For the present study, two different and independent testing approaches have been adopted:

- Validation against artificially created data randomly developed from specific probability density functions, testing the performance of the proposed methodology in an ideal framework.
- Evaluation in real world conditions against observations of meteorlogical stations in Greece and the Aegean Sea.

The first approach is based on idealized data randomly created which, however, are coming from the real distributions that describe wind speed values over specific sites with the associated "error" considered as Gaussian. More precisely, the following procedure has been followed:

- The distribution that optimally describes the real records from a meteorological station is defined.
- Idealized observations are randomly created by the obtained probability density function (pdf).
- The corresponding "model" values are considered to diverge from the idealized observations by a randomly created noise from the normal distribution

In the second – real world – approach, observations and corresponding forecasts from the Skiron model are employed over a number of sites covering all the latitude range of Greece as presented in Figure 1. The available wind speed time series cover a period of 3 years during which independent test periods have been selected corresponding to different seasonal characteristics.

FIGURE 1. *The sites of test cases.*

The statistical indexes

The evaluation of the results has been based on the following statistical indexes:

$$\textbf{Bias: } \frac{1}{N}\sum_{i=1}^{N}\left(obs(i) - mod(i)\right),$$

where *obs* and *mod* stand for the observations and modeled values respectively while *N* gives the size of the sample under study. This is a basic but particularly important measure of success for any optimization system and especially Kalman type filters.

$$\textbf{Root Mean Square Error: } \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(obs(i) - mod(i)\right)^2}$$

a measure of the variation of the model discrepancies.

The **Nash-Sutcliffe model efficiency coefficient (NS):**

$$NS = 1 - \frac{\sum (X_{obs,i} - X_{model})^2}{\sum (X_{obs,i} - \overline{X_{obs}})^2}$$

where X_{obs} is observed values and X_{model} is modelled values at time/place *i*.

This is a commonly used index to assess the models predictive accuracy. Nash-Sutcliffe efficiencies can range from -infinity to 1, where an efficiency of 1 corresponds to a perfect match between model and observations and an efficiency of 0 indicates that the model predictions are as accurate as the usage of the mean of the observed data as prediction value.

Finally, the Percentiles, that is the values under which specific percentiles of the data set under study are recorded, of the modeled and observed data are utilized for comparing the full corresponding distributions and revealing discrepancies both in the most common appearing cases as well as to non-frequent/extremes values that could be of great importance for applications.

Analysis of indicable results

Beginning with the analysis of the results, in Figure 2, the above statistical measures are presented for the site of Skyros Island in the Agean Sea for the application of a classical/standard Kalman filtering and the new proposed IG-Kalman filter over idealized data. The latter have been randomly produced based on the Weibull distribution with scale parameter 1.67 and shape 7.10, which have been estimated by all the available real observations in the area.

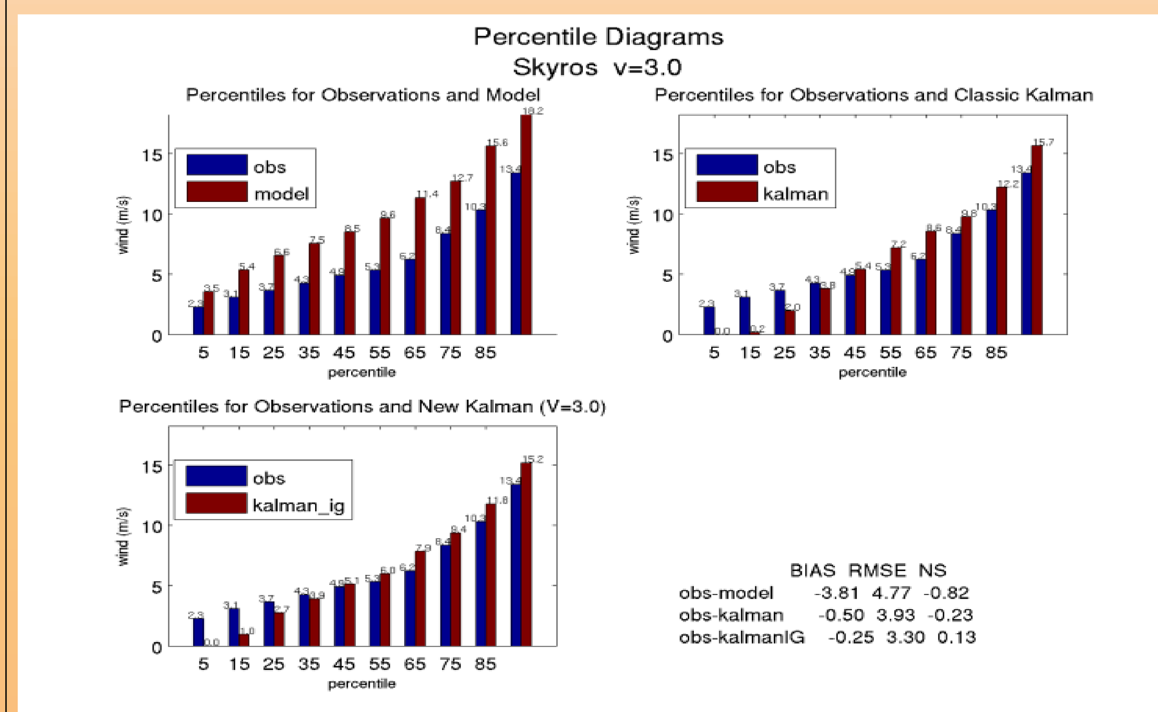


Figure 2. The Error statistical indexes and the percentiles of the distributions of the observations, direct model outputs, a classical Kalman filter and the new IG-Kalman filter for the Skyros station based on idealized data analysis.

The obtained results for the proposed new IG-Kalman filter are more than satisfactory leading to the significant reduction of the Bias, both of the direct model outputs and the standard Kalman filter, and to a noticeable improvement of RMSE and NS indexes, facts that prove the general qualitative improvement of the final filtered forecasts. This is further underline by the convergence of the percentile values in all their range for observations and IG-Kalman filtered predictions.

Similarly good results have been obtained over all the stations under study in the ideal environment of artificially produced data based on the probability distribution functions that optimally fit the history data of each station. A second indicative example is presented in Figure 3 for the sear of Souda in Crete island. Artificially idealized data have been tested also in this case randomly obtained by the Weibull d

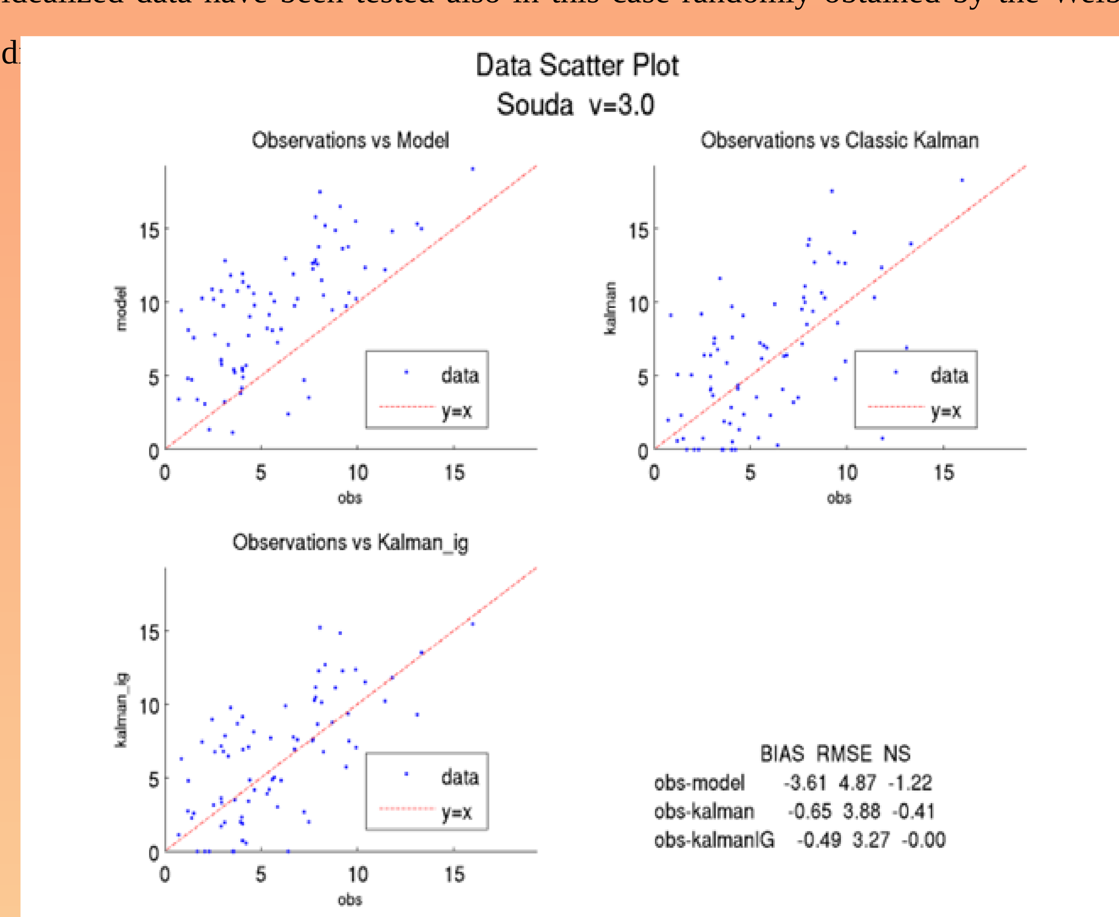
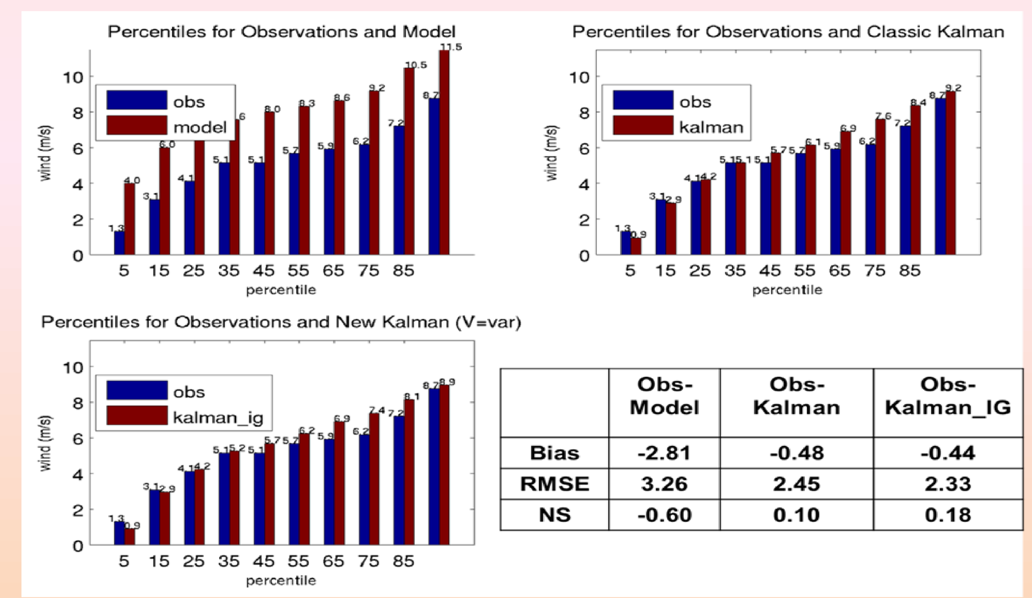


Figure 3. The Error statistical indexes and scatter plots of the observations, direct model outputs, a classical Kalman filter and the new IG-Kalman filter for the Souda station based on idealized data analysis.

The proposed optimization methodology succeeded to reduce drastically the initial model overestimation and to limit the variability of the remaining non systematic bias.

In the following, always indicative, test cases, we are focusing on applications of the proposed IG-Kalman filter on real NWP modeld wind speed forecasts and the corresponding observations, as described in the Methodology Section.

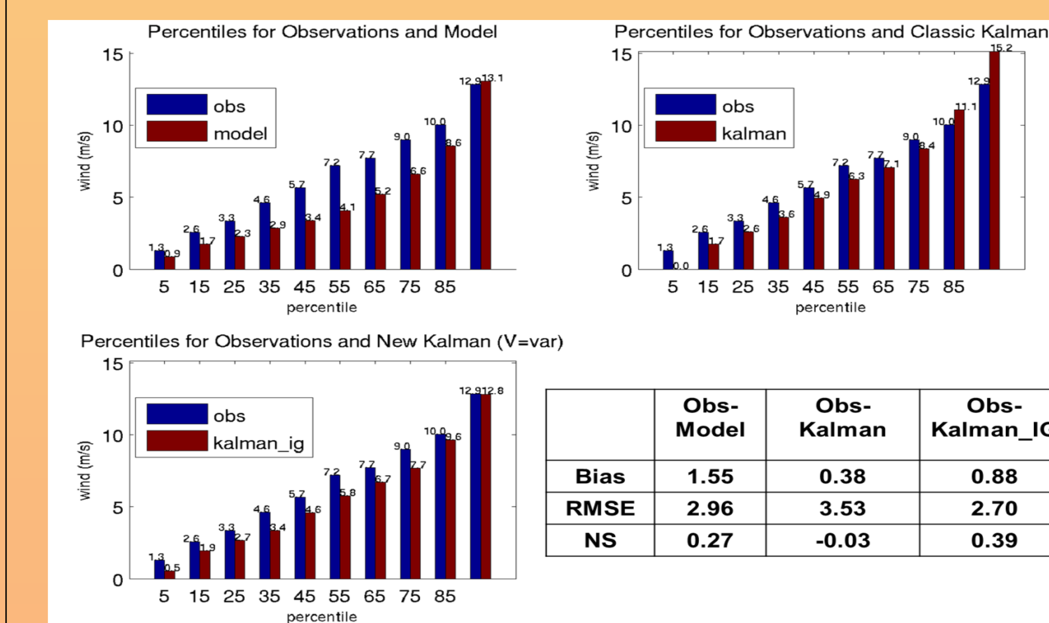
In Figure 4, an indicative analysis of the "real world" applications is presented. The new Information Geometry Kalman filter has been utilized for improving the Numerical Weather Prediction model outputs leading to very satisfactory results that almost eliminate the initial model's overestimations even for the high/extreme wind speed values and reduce at percentage of almost 30% the error variability as estimated by the RMSE values and, therefore, the corresponding uncertainty of forecasted values. The IG-Kalman filters seems also to outperform the classical Kalman filter employed.



	Obs-Model	Obs-Kalman	Obs-Kalman_IG
Bias	-2.81	-0.48	-0.44
RMSE	3.26	2.45	2.33
NS	-0.60	0.10	0.18

Figure 4. The Error statistical indexes and the percentiles of the distributions of the observations, direct model outputs, a classical Kalman filter and the new IG-Kalman filter for the Chios station based on NWP modeled and corresponding observations.

Similarly good is the transformation of the distribution of the IG-Kalman filtered predictions for the station of Mykonos island where the case of high speed values (95 percentile) is noticeably improved in contrast to the classical Kalman filter application. This fact is especially important for wind farm site assessment since most of the wind power generators have a cut of frequency above which they totally stop or limit their production. The statistical measures are also significantly improved here.



	Obs-Model	Obs-Kalman	Obs-Kalman_IG
Bias	1.55	0.38	0.88
RMSE	2.96	3.53	2.70
NS	0.27	-0.03	0.39

Figure 5. The Error statistical indexes and the percentiles of the distributions of the observations, direct model outputs, a classical Kalman filter and the new IG-Kalman filter for the Mykonos station based on NWP modeled and corresponding observations.

Conclusions

The analysis of idealized wind speed data indicates an overall improvement of model predictions using the proposed Kalman IG filter. In the same direction, the study of real data results confirms the level of improvement presented in idealized cases. In cases where the original-direct model outputs are of a very good quality leaving limited space for improvements both the classical Kalman and Kalman_IG filters provide similar level of improvements. More over , the proposed filter practically eliminates systematic error in the vast majority of idealized cases. In real tests the error is reduced by at least 50% and reaching a 90-100% rate of improvement in a significant number of cases tested.

Except BIAS statistical index, the level of improvement concerning RMSE and Nash-Sutcliffe coefficient was calculated. In idealized tests, both RMSE and NS indices were improved, while NS displayed a significant rate of correction of more than 50%. The improvement to RMSE reached 20-40% in all cases. The aforementioned behavior was repeated at real data where in the 86% of the test cases RMSE and NS coefficient were improved at an average rate of 15% and 80% respectively.

The results concerning significant wave height correction using classic Kalman filter and Kalman IG present a slightly different behavior than wind speed data due to their particularly low values of variance. Nevertheless, the use of a constant variance value of 0.001 offers the same level and quality of corrections to wave data as those achieved by classic Kalman.

As a conclusion, the main target of the proposed Kalman IG filter and Kalman filters in general is the elimination of systematic errors. In the present analysis, systematic errors are quantified by the statistical index BIAS, which is significantly improved by Kalman IG filter. The correction achieved in other statistical parameters such as RMSE and NS, confirms the ability of the proposed filter to reduce the variability of the error.

Finally, as for another improvement feature, the amount of CPU time needed to complete the execution of both filters, Kalman IG and classic Kalman, were recorded during the runs of all test cases, both idealized and real. There is a reduce in average CPU time of 22.9 % in the case of the runs of new Kalman IG filter compare to runs of the classical Kalman filter, indicating the potential of the proposed filter to correct model predictions accurately and reduce the needed CPU resources.

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