A NEW MATHEMATICAL FRAMEWORK FOR THE STUDY AND OPTIMAZATION OF WIND/WAVE FORECASTS

A ***n-dimensional statistical manifold*** is a family of probability distributions

$$S = \{p_\xi = p(x;\xi) \mid \xi = [\xi_1, \xi_2, \ldots, \xi_n] \in \Xi\}$$

where each element may be parameterized using n real valued variables in an open subset Ξ of $\mathbb{R}^n$ while the mapping $\xi \rightarrow p_\xi$ is injective and smooth.

The geometrical framework in a statistical manifold is characterized by the ***Fisher information matrix*** which at a point ξ is a nxn matrix $G(\xi) = [g_{ij}(\xi)]$, with elements

$$g_{ij}(\xi) = E_{x|\xi}[\partial_i \ell(x;\xi) \partial_j \ell(x;\xi)] = \int \partial_i \ell(x;\xi) \partial_j \ell(x;\xi) p(x;\xi) dx\,,$$

where $\ell(x;\xi) = log[p(x;\xi)]$ and

$$E_{x|\xi}[f] = \int f(x) p(x;\xi) dx$$

is the expectation with respect to the distribution p.

The Fisher information matrix $G(\xi)$ is symmetric and positive semi-definite.

If $G(\xi)$ is positive definite, then a Riemannian metric can be defined on the statistical manifold corresponding to the inner product induced on the natural basis of the coordinate system $[\xi_i]$:

$$g_{ij} = \langle \partial_i | \partial_j \rangle$$

This Riemannian metric is called the ***Fisher metric*** or the ***information metric***.

The corresponding geometric properties are characterized by the *Christoffel symbols* (Γ^i_{jk}) of the *Levi-Civita connection* with respect to the Fisher metric defined by the relations:

$$\Gamma_{jk,h}(\xi) = E_\xi\left[\left(\partial_j \partial_k \ell_\xi + \frac{1}{2} \partial_j \ell_\xi \partial_k \ell_\xi\right)\left(\partial_h \ell_\xi\right)\right], \quad i,\ j\ = 1,\ 2, \ldots, n,$$

$$\Gamma_{jk,h} = \sum_{i=1}^{2} g_{hi} \Gamma^i_{jk} \text{ (h=1, 2).}$$

The minimum distance between two elements f_1 *and* f_2 of a statistical manifold S is defined by the corresponding geodesic which is the minimum length curve ω that connects them. Such a curve

$$\omega = (\omega_i) \; : \; \mathbb{R} \rightarrow S$$

is the solution of a system of 2nd order differential equations:

$$\omega_i''(t) + \sum_{j,k=1}^{n} \Gamma_{jk}^{i}(t)\, \omega'_j(t)\omega'_k(t) = 0, \qquad i = 1, 2, \ldots, n.$$

under the conditions $\omega(0) = f_1, \;\; \omega(1) = f_2$.

In the special case of a flat/Euclidean space, the above system reduces to

$$\omega'' = 0$$

that leads to straight lines and is associated with least square approaches.

SITES AND DATA UNDER STUDY

The data obtained within the framework of work package 1 cover the major area of Greece and correspond to different climatological characteristics (Table 1, Figure 1). Three different sources of data were used:

- Observations from buoy measurements
- Hindcast data (model simulations in which any available observations are assimilated) from the FP7 program MARINA Platform project (http://forecast.uoa.gr/proj_marina.php).
- Operational forecasts from the wind and wave modeling system of the Atmospheric Modeling and Weather Forecasting group (AM&WFG), Physics Department, University of Athens.

The meteorological parameters under study are these that directly affecting the wind/wave energy potential:

- Wind speed
- Significant wave height.

Figure 1: Sites under study in Greece

Wind Speed (Aegean Sea)	Wind Speed (Ionian Sea)	Significant wave Height (Aegean Sea)
Skyros	Argostoli	Crete
Chios	Kerkyra	Lesvos
Mykonos		Mykonos
Milos		
Thira		
Souda		
Siteia		

Table 1: Sites in the Aegean Sea and in the Ionian Sea

MODELS USED

The Atmospheric Model

The SKIRON modeling system (Kallos 1997, Papadopoulos et al. 2001, Spyrou et al., 2010) was utilized for the present study. SKIRON is a non-hydrostatic atmospheric model developed at the University of Athens by the Atmospheric Modeling and Weather Forecasting Group, in the basis of the Eta/NCEP model (Janjic, 1994). As initial conditions the model uses fields from the NCEP/GFS model (National Center for Environmental Prediction/Global Forecast System model) and SST (Sea Surface Temperature) data at a resolution of 0.5°. Vegetation and topography data are applied at a resolution of 30'' and soil texture data at 2'.

The model covers a domain that includes Europe, North Africa, Middle East and the entire Mediterranean region with a horizontal increment of 0.05°x0.05°. In the vertical, 45 Eta levels are used from the ground to the model's top.

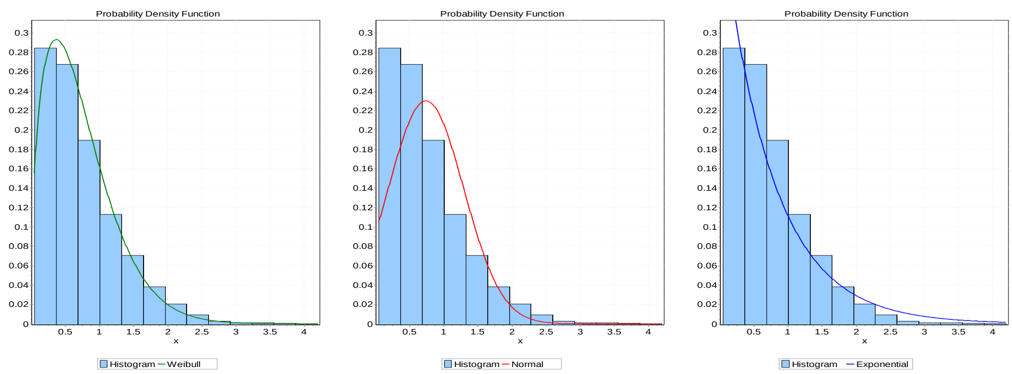
The Wave Model

The wave system is based on the wave model WAM-ECMWF version, CY33R1 (WAMDIG, 1988; Komen et al., 1994, Bidlot and Janssen, 2003), a third generation wave model that has been adopted by several operational and research centers worldwide and is recognized as one of the most credible wave models today. WAM solves the wave transport equation explicitly without any presumptions on the shape of the wave spectrum using the full set of degrees of freedom of a 2d wave spectrum. The model runs for any given regional or global grid with a prescribed bathymetric dataset. The grid resolution can be arbitrary in space and time. The propagation can be done on a latitudinal – longitudinal or on a Cartesian grid. WAM is able to run for deep and shallow water and includes the effect of wave refraction from changes in depth and from ocean currents.

STATISTICAL MANIFOLDS AND GEOMETRICAL PROPERTIES

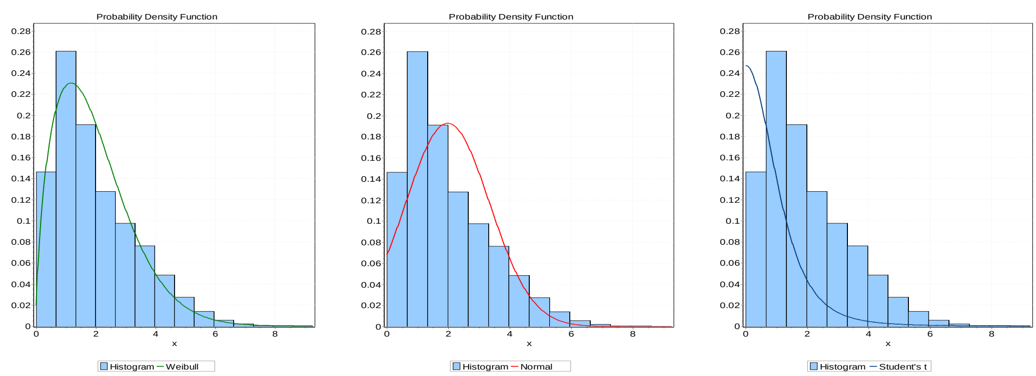
For each area under study the probability density functions that optimally fit the three data sets obtained (observations, MARINA project's results, Operational forecasts) are defined. A variety of distributions have been tested (Logistic, Normal, Gamma, Log-Gamma, Log-Logistic, Lognormal, Weibull, Generalized Logistic) at several levels of statistical significance by utilizing different fitting tests (Kolmogorov-Smirnov, Anderson-Darling).

In all cases the Weibull distribution seems to prevail. Indicative examples are given in the next two figures where the suitability of the Weibull distribution against other commonly used probability density functions (Normal, Exponential, Students) is evident:



(a) *(b)* *(c)*

Figure 2. Fitting results of the probability density functions Weibull (a), Normal (b) and Exponential (c) for the significant wave height modeled in Lesvos site.



(a) *(b)* *(c)*

Figure 3. Fitting results of the probability density functions Weibull (a), Normal (b) and Students (c) for the wind speed data in Mykonos site.

In the next part of this section and foe each site of interest the following information is provided:

- The optimum distribution and the corresponding scale/shape parameters
- The Fisher inner product matrix that defines the geometrical environment
- The form of the corresponding geodesics that is the minimum length curves.

The **Weibull distribution** optimally fits the data under study which are recognized as elements of the Weibull statistical manifold

$$S = \left\{ f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}},\ \alpha,\, \beta > 0 \right\}$$

The Fisher information matrix takes the form $G(\alpha,\beta) = \begin{bmatrix} \alpha^2\beta^2 & \beta(1-\gamma) \\ \beta(1-\gamma) & \frac{6(\gamma-1)^2+\pi^2}{6\alpha^2} \end{bmatrix}$ where γ is the Euler Gamma.

The corresponding geodesics, necessary to estimate distances between different data sets are solutions of the second order system.

$$\omega_1''(t) + \frac{6\left(\gamma a - a - \frac{\pi^2}{6}\right)}{\pi^2\beta}(\omega'_1(t))^2 + \frac{12\left(\gamma^2 - 2\gamma + \frac{\pi^2}{6} + 1\right)}{\pi^2 a}\omega'_1(t)\omega'_2(t) - \frac{6(1-\gamma)\beta\left(\gamma^2 - 2\gamma + \frac{\pi^2}{6} + 1\right)}{\pi^2 a^3}(\omega'_2(t))^2 = 0$$

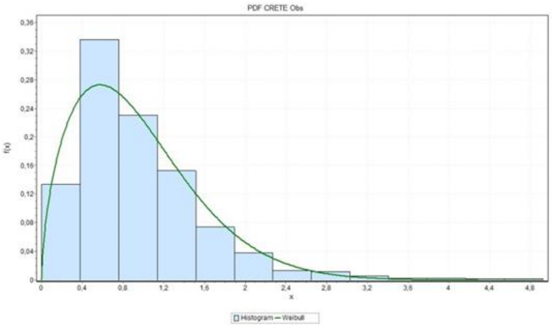
$$\omega_2''(t) - \frac{a^3}{\pi^2\beta^2}(\omega'_1(t))^2 + \frac{12a(1-\gamma)}{\pi^2\beta}\omega'_1(t)\omega'_2(t) - \frac{6\left(\gamma^2 - 2\gamma + \frac{\pi^2}{6} + 1\right)}{\pi^2 a}(\omega'_2(t))^2 = 0,$$

RESULTS

For each area under study we present the final results.

CRETE (Significant Wave Height):

- Observations



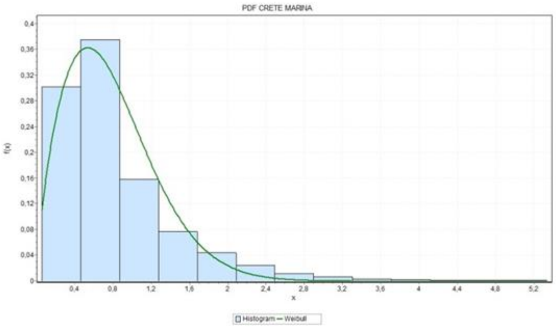
Distribution: *Weibull*
Shape parameter: $\alpha = 1.6$
Scale parameter: $b = 1.5$

Fisher information matrix: $G = \begin{bmatrix} 5.76 & 0.63 \\ 0.63 & 0.71 \end{bmatrix}$

Geodesics equation system:
$\omega_1'' - 0.94(\omega_1')^2 + 1.39\omega_1'\omega_2' - 0.17(\omega_2')^2 = 0$
$\omega_2'' - 0.18(\omega_1')^2 + 0.55\omega_1'\omega_2' - 0.69(\omega_2')^2 = 0$

Figure 4a: Distribution fitting for the observations of SWH at Crete

- MARINA



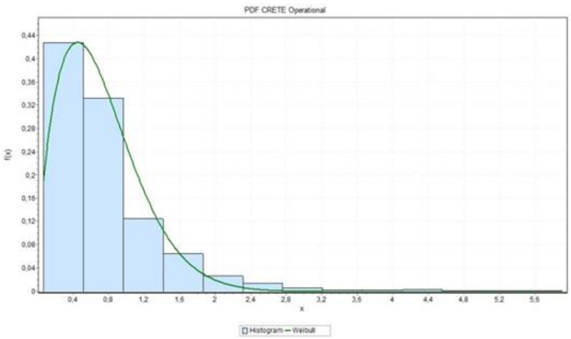
Distribution: *Weibull*
Shape parameter: $\alpha = 1.73$
Scale parameter: $b = 0.88$

Fisher information matrix: $G = \begin{bmatrix} 2.32 & 0.37 \\ 0.37 & 0.61 \end{bmatrix}$

Geodesics equation system:
$\omega_1'' - 1.64(\omega_1')^2 + 1.28\omega_1'\omega_2' - 0.08(\omega_2')^2 = 0$
$\omega_2'' - 0.68(\omega_1')^2 + 1.01\omega_1'\omega_2' - 0.64(\omega_2')^2 = 0$

Figure 4b: Distribution fitting for the MARINA project SWH modeled data at Crete

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 1.63$
Scale parameter: $b = 0.8$

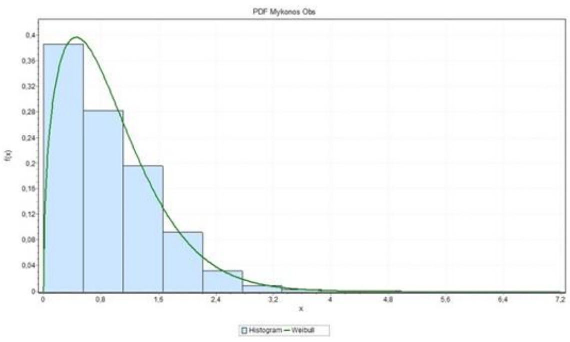
Fisher information matrix: $G = \begin{bmatrix} 1.70 & 0.34 \\ 0.34 & 0.69 \end{bmatrix}$

Geodesics equation system:
$\omega_1'' - 1.77(\omega_1')^2 + 1.36\omega_1'\omega_2' - 0.09(\omega_2')^2 = 0$
$\omega_2'' - 0.69(\omega_1')^2 + 1.05\omega_1'\omega_2' - 0.68(\omega_2')^2 = 0$

Figure 4c: Distribution fitting for the operational model SWH results at Crete

MYKONOS (Significant Wave Height):

- Observations



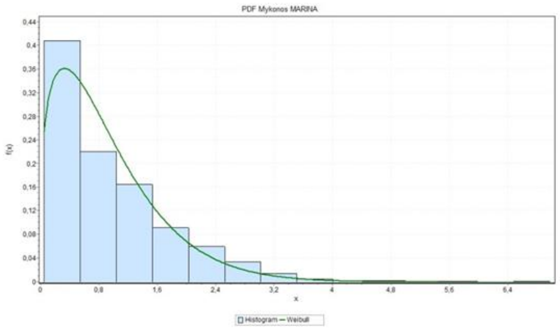
Distribution: *Weibull*
Shape parameter: $\alpha = 1.44$
Scale parameter: $b = 1.03$

Fisher information matrix: $G = \begin{bmatrix} 2.20 & 0.44 \\ 0.44 & 0.88 \end{bmatrix}$

Geodesics equation system:
$\omega_1'' - 1.33(\omega_1')^2 + 1.54\omega_1'\omega_2' - 0.16(\omega_2')^2 = 0$
$\omega_2'' - 0.29\,(\omega_1')^2 + 0.72\omega_1'\omega_2' - 0.77(\omega_2')^2 = 0$

Figure 5a: Distribution fitting for the observations of SWH at Mykonos

- MARINA



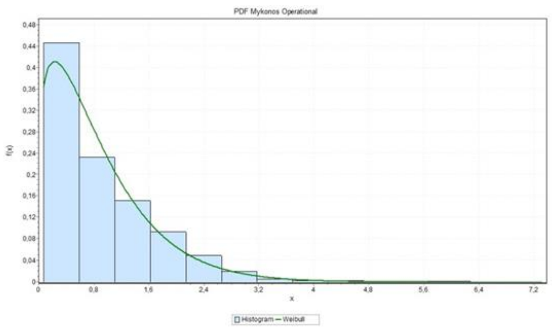
Distribution: *Weibull*
Shape parameter: $\alpha = 1.30$
Scale parameter: $b = 1.00$

Fisher information matrix: $G = \begin{bmatrix} 1.69 & 0.42 \\ 0.42 & 1.08 \end{bmatrix}$

Geodesics equation system:
$\omega_1'' - 1.33(\omega_1')^2 + 1.71\omega_1'\omega_2' - 0.21(\omega_2')^2 = 0$
$\omega_2'' - 0.22(\omega_1')^2 + 0.67\omega_1'\omega_2' - 0.85(\omega_2')^2 = 0$

Figure 5b: Distribution fitting for the MARINA project SWH modeled data at Mykonos

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 1.21$
Scale parameter: $b = 0.94$

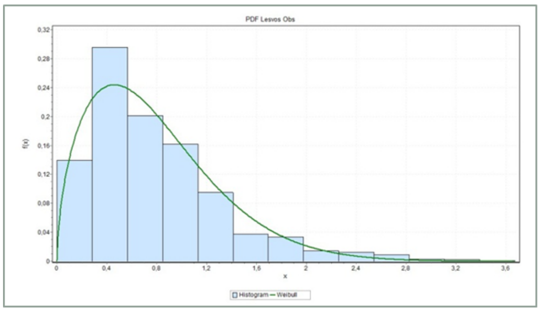
Fisher information matrix: $G = \begin{bmatrix} 1.29 & 0.40 \\ 0.40 & 1.25 \end{bmatrix}$

Geodesics equation system:
$\omega_1'' - 1.39(\omega_1')^2 + 1.83\omega_1'\omega_2' - 0.25(\omega_2')^2 = 0$
$\omega_2'' - 0.20(\omega_1')^2 + 0.66\omega_1'\omega_2' - 0.92(\omega_2')^2 = 0$

Figure 5c: Distribution fitting for the operational model SWH results at Mykonos

<u>LESVOS (Significant Wave Height):</u>

- Observations



Distribution: *Weibull*
Shape parameter: $\alpha = 1.57$
Scale parameter: $b = 0.87$

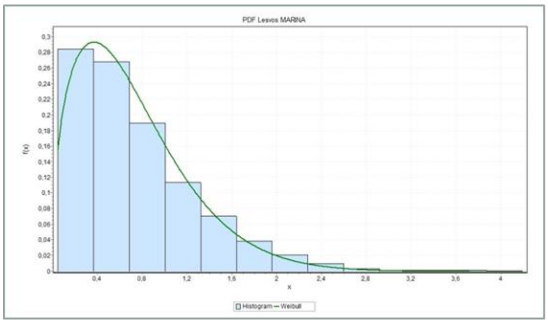
Fisher information matrix: $G = \begin{bmatrix} 1.87 & 0.37 \\ 0.37 & 0.74 \end{bmatrix}$

Geodesics equation system:

$$\omega_1'' - 1.61(\omega_1')^2 + 1.41\omega_1'\omega_2' - 0.11(\omega_2')^2 = 0$$
$$\omega_2'' - 0.52(\omega_1')^2 + 0.93\omega_1'\omega_2' - 0.71(\omega_2')^2 = 0$$

Figure 6a: Distribution fitting for the observations of SWH at Lesvos

- MARINA



Distribution: *Weibull*
Shape parameter: $\alpha = 1.46$
Scale parameter: $b = 0.81$

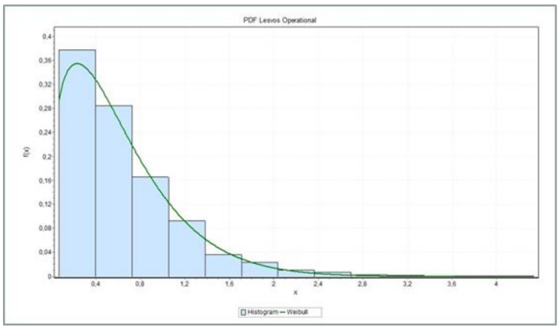
Fisher information matrix: $G = \begin{bmatrix} 1.40 & 0.34 \\ 0.34 & 0.86 \end{bmatrix}$

Geodesics equation system:

$$\omega_1'' - 1.70(\omega_1')^2 + 1.52\omega_1'\omega_2' - 0.12(\omega_2')^2 = 0$$
$$\omega_2'' - 0.48(\omega_1')^2 + 0.93\omega_1'\omega_2' - 0.76(\omega_2')^2 = 0$$

Figure 6b: Distribution fitting for the MARINA project SWH modeled data at Lesvos

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 1.32$
Scale parameter: $b = 0.68$

Fisher information matrix: $G = \begin{bmatrix} 0.81 & 0.29 \\ 0.29 & 1.05 \end{bmatrix}$

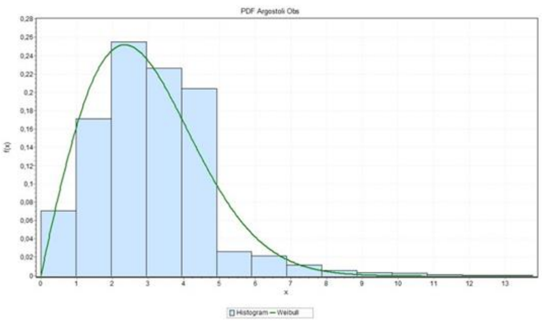
Geodesics equation system:

$$\omega_1'' - 1.97(\omega_1')^2 + 1.68\omega_1'\omega_2' - 0.14(\omega_2')^2 = 0$$
$$\omega_2'' - 0.50(\omega_1')^2 + 1.00\omega_1'\omega_2' - 0.84(\omega_2')^2 = 0$$

Figure 6c: Distribution fitting for the operational model SWH results at Lesvos

<u>ARGOSTOLI (Wind Speed):</u>

- Observations



Distribution: *Weibull*
Shape parameter: $\alpha = 1.98$
Scale parameter: $b = 3.33$

Fisher information matrix: $G = \begin{bmatrix} 43.47 & 1.41 \\ 1.41 & 0.47 \end{bmatrix}$

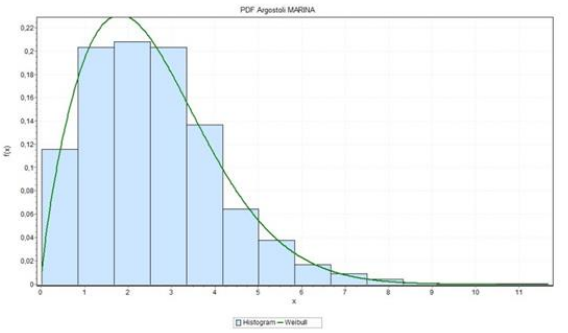
Geodesics equation system:

$\omega_1'' - 0.45(\omega_1')^2 + 1.12\omega_1'\omega_2' - 0.20(\omega_2')^2 = 0$

$\omega_2'' - 0.07(\omega_1')^2 + 0.31\omega_1'\omega_2' - 0.56(\omega_2')^2 = 0$

Figure 7a: Distribution fitting for the observations of wind speed at Argostoli

- MARINA



Distribution: *Weibull*
Shape parameter: $\alpha = 1.78$
Scale parameter: $b = 2.88$

Fisher information matrix: $G = \begin{bmatrix} 26.28 & 1.22 \\ 1.22 & 0.58 \end{bmatrix}$

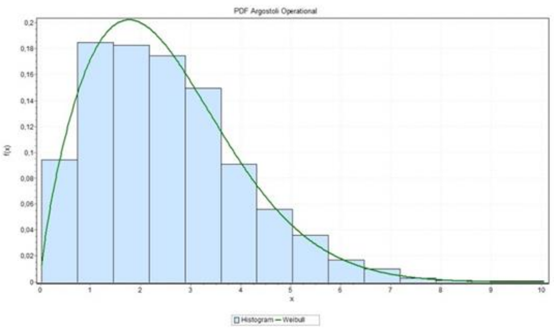
Geodesics equation system:

$\omega_1'' - 0.51(\omega_1')^2 + 1.25\omega_1'\omega_2' - 0.24(\omega_2')^2 = 0$

$\omega_2'' - 0.07(\omega_1')^2 + 0.32\omega_1'\omega_2' - 0.62(\omega_2')^2 = 0$

Figure 7b: Distribution fitting for the MARINA project wind speed modeled data at Argostoli

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 1.77$
Scale parameter: $b = 2.82$

Fisher information matrix: $G = \begin{bmatrix} 24.91 & 1.19 \\ 1.19 & -0.58 \end{bmatrix}$

Geodesics equation system:

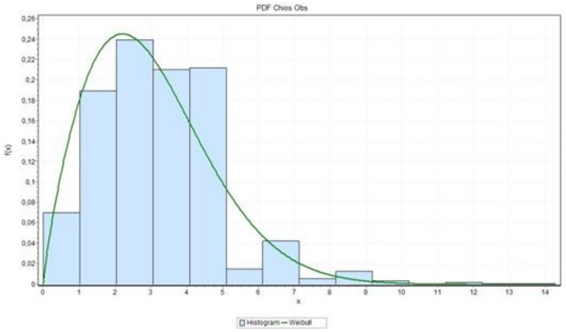
$\omega_1'' - 0.52(\omega_1')^2 + 1.25\omega_1'\omega_2' - 0.24(\omega_2')^2 = 0$

$\omega_2'' - 0.07(\omega_1')^2 + 0.32\omega_1'\omega_2' - 0.63(\omega_2')^2 = 0$

Figure 7c: Distribution fitting for the operational model wind speed results at Argostoli

CHIOS (Wind Speed):

- Observations



Distribution: *Weibull*
Shape parameter: $\alpha = 1.83$
Scale parameter: $b = 3.38$

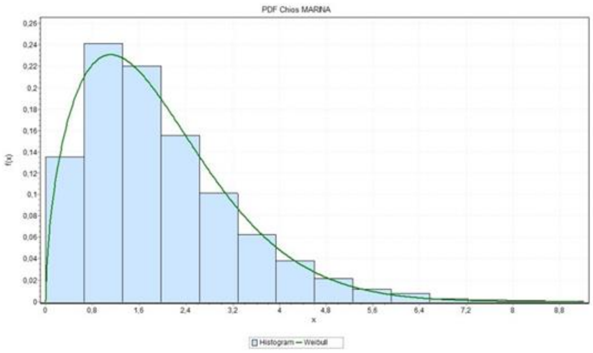
Fisher information matrix: $G = \begin{bmatrix} 38.26 & 1.43 \\ 1.43 & 0.54 \end{bmatrix}$

Geodesics equation system:

$$\omega_1'' - 0.44(\omega_1')^2 + 1.21\omega_1'\omega_2' - 0.26(\omega_2')^2 = 0$$
$$\omega_2'' - 0.05(\omega_1')^2 + 0.28\omega_1'\omega_2' - 0.61(\omega_2')^2 = 0$$

Figure 8a: Distribution fitting for the observations of wind speed at Chios

- MARINA



Distribution: *Weibull*
Shape parameter: $\alpha = 1.55$
Scale parameter: $b = 2.14$

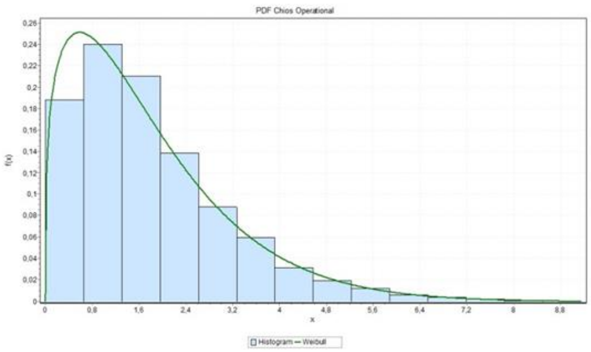
Fisher information matrix: $G = \begin{bmatrix} 11.00 & 0.91 \\ 0.91 & 0.76 \end{bmatrix}$

Geodesics equation system:

$$\omega_1'' - 0.65(\omega_1')^2 + 1.43\omega_1'\omega_2' - 0.27(\omega_2')^2 = 0$$
$$\omega_2'' - 0.08(\omega_1')^2 + 0.37\omega_1'\omega_2' - 0.72(\omega_2')^2 = 0$$

Figure 8b: Distribution fitting for the MARINA project wind speed modeled data at Chios

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 1.28$
Scale parameter: $b = 1.92$

Fisher information matrix: $G = \begin{bmatrix} 6.04 & 0.81 \\ 0.81 & 1.11 \end{bmatrix}$

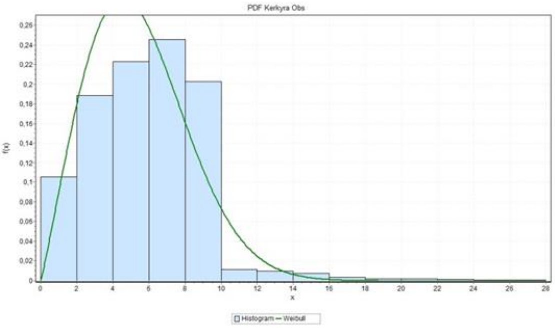
Geodesics equation system:

$$\omega_1'' - 0.69(\omega_1')^2 + 1.73\omega_1'\omega_2' - 0.43(\omega_2')^2 = 0$$
$$\omega_2'' - 0.06(\omega_1')^2 + 0.34\omega_1'\omega_2' - 0.87(\omega_2')^2 = 0$$

Figure 8c: Distribution fitting for the operational model wind speed results at Chios

KERKYRA (Wind Speed):

- Observations



Distribution: *Weibull*
Shape parameter: $\alpha = 2.10$
Scale parameter: $b = 6.16$

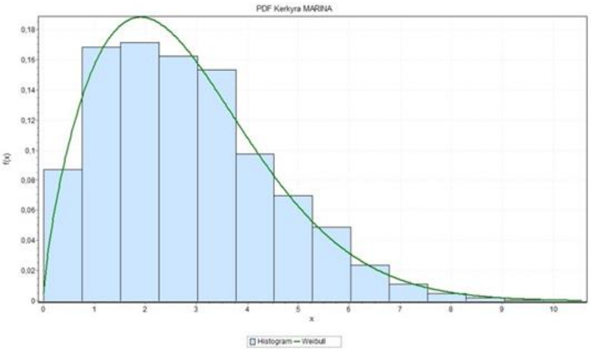
Fisher information matrix: $G = \begin{bmatrix} 167.34 & 2.61 \\ 2.61 & 0.41 \end{bmatrix}$

Geodesics equation system:

$$\omega_1'' - 0.25(\omega_1')^2 + 1.06\omega_1'\omega_2' - 0.31(\omega_2')^2 = 0$$
$$\omega_2'' - 0.02(\omega_1')^2 + 0.18\omega_1'\omega_2' - 0.53(\omega_2')^2 = 0$$

Figure 9a: Distribution fitting for the observations of wind speed at Kerkyra

- MARINA



Distribution: *Weibull*
Shape parameter: $\alpha = 1.72$
Scale parameter: $b = 3.14$

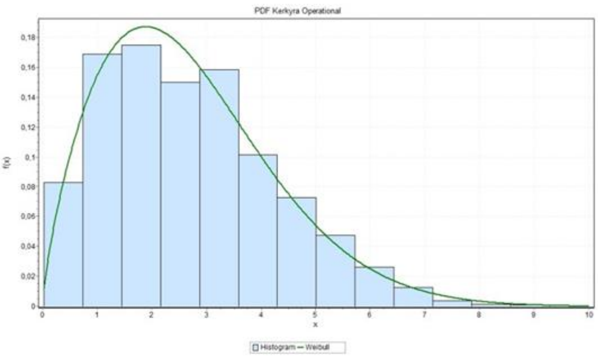
Fisher information matrix: $G = \begin{bmatrix} 29.17 & 1.33 \\ 1.33 & 0.62 \end{bmatrix}$

Geodesics equation system:

$$\omega_1'' - 0.46(\omega_1')^2 + 1.29\omega_1'\omega_2' - 0.29(\omega_2')^2 = 0$$
$$\omega_2'' - 0.05(\omega_1')^2 + 0.28\omega_1'\omega_2' - 0.64(\omega_2')^2 = 0$$

Figure 9b: Distribution fitting for the MARINA project wind speed modeled data at Kerkyra

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 1.76$
Scale parameter: $b = 3.03$

Fisher information matrix: $G = \begin{bmatrix} 28.44 & 1.28 \\ 1.28 & 0.59 \end{bmatrix}$

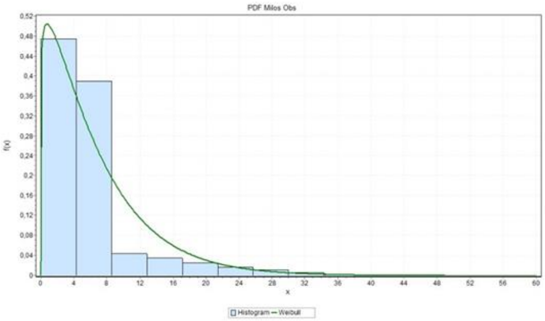
Geodesics equation system:

$$\omega_1'' - 0.48(\omega_1')^2 + 1.26\omega_1'\omega_2' - 0.26(\omega_2')^2 = 0$$
$$\omega_2'' - 0.06(\omega_1')^2 + 0.30\omega_1'\omega_2' - 0.63(\omega_2')^2 = 0$$

Figure 9c: Distribution fitting for the operational model wind speed results at Kerkyra

MILOS (Wind Speed):

- Observations



Distribution: *Weibull*
Shape parameter: $\alpha = 1.09$
Scale parameter: $b = 6.89$

Fisher information matrix: $G = \begin{bmatrix} 56.40 & 2.91 \\ 2.91 & 1.54 \end{bmatrix}$

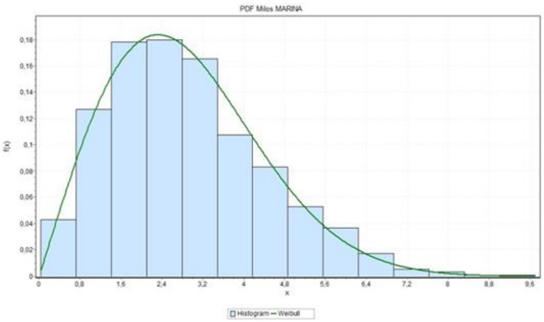
Geodesics equation system:

$\omega_1'' - 0.19(\omega_1')^2 + 2.03\omega_1'\omega_2' - 2.50(\omega_2')^2 = 0$

$\omega_2'' - 0.00(\omega_1')^2 + 0.08\omega_1'\omega_2' - 1.02(\omega_2')^2 = 0$

Figure 10a: Distribution fitting for the observations of wind speed at Milos

- MARINA



Distribution: *Weibull*
Shape parameter: $\alpha = 2.02$
Scale parameter: $b = 3.25$

Fisher information matrix: $G = \begin{bmatrix} 43.10 & 1.37 \\ 1.37 & 0.45 \end{bmatrix}$

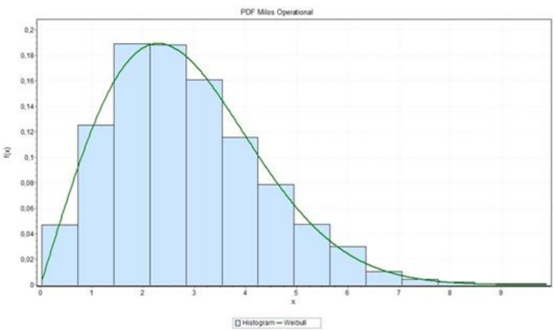
Geodesics equation system:

$\omega_1'' - 0.47(\omega_1')^2 + 1.10\omega_1'\omega_2' - 0.18(\omega_2')^2 = 0$

$\omega_2'' - 0.08(\omega_1')^2 + 0.32\omega_1'\omega_2' - 0.55(\omega_2')^2 = 0$

Figure 10b: Distribution fitting for the MARINA project wind speed modeled data at Milos

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 2.02$
Scale parameter: $b = 3.21$

Fisher information matrix: $G = \begin{bmatrix} 42.04 & 1.36 \\ 1.36 & 0.45 \end{bmatrix}$

Geodesics equation system:

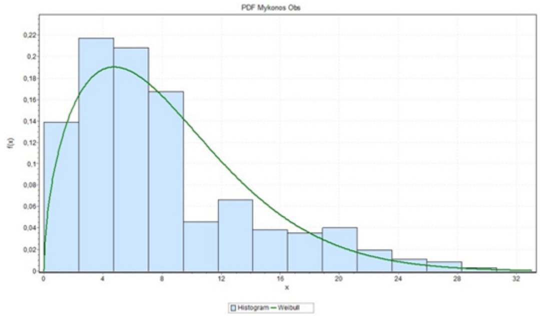
$\omega_1'' - 0.47(\omega_1')^2 + 1.10\omega_1'\omega_2' - 0.18(\omega_2')^2 = 0$

$\omega_2'' - 0.08(\omega_1')^2 + 0.32\omega_1'\omega_2' - 0.55(\omega_2')^2 = 0$

Figure 10c: Distribution fitting for the operational model wind speed results at Milos

MYKONOS (Wind Speed):

- Observations



Distribution: *Weibull*
Shape parameter: $\alpha = 1.54$
Scale parameter: $b = 9.32$

Fisher information matrix: $G = \begin{bmatrix} 206.00 & 3.94 \\ 3.94 & 0.77 \end{bmatrix}$

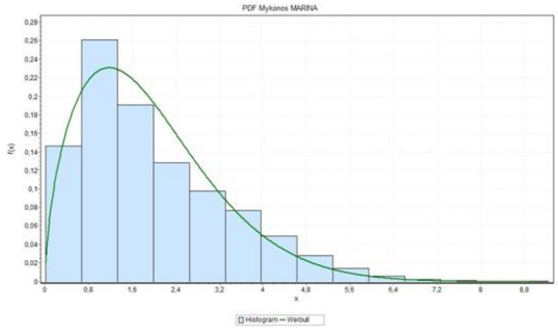
Geodesics equation system:

$\omega_1'' - 0.15(\omega_1')^2 + 1.44\omega_1'\omega_2' - 1.20(\omega_2')^2 = 0$

$\omega_2'' - 0.00(\omega_1')^2 + 0.08\omega_1'\omega_2' - 0.72(\omega_2')^2 = 0$

Figure 11a: Distribution fitting for the observations of wind speed at Mykonos

- MARINA



Distribution: *Weibull*
Shape parameter: $\alpha = 1.59$
Scale parameter: $b = 2.17$

Fisher information matrix: $G = \begin{bmatrix} 11.90 & 0.92 \\ 0.92 & 0.72 \end{bmatrix}$

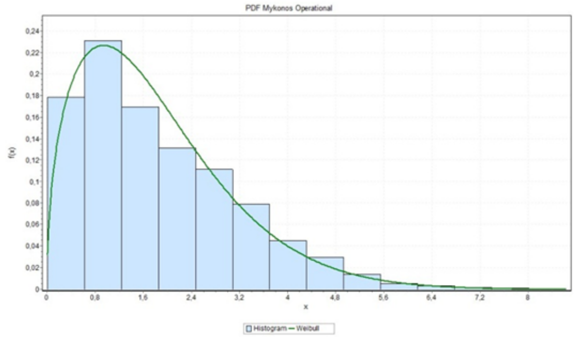
Geodesics equation system:

$\omega_1'' - 0.65(\omega_1')^2 + 1.39\omega_1'\omega_2' - 0.25(\omega_2')^2 = 0$

$\omega_2'' - 0.09(\omega_1')^2 + 0.38\omega_1'\omega_2' - 0.70(\omega_2')^2 = 0$

Figure 11b: Distribution fitting for the MARINA project wind speed modeled data at Mykonos

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 1.47$
Scale parameter: $b = 2.01$

Fisher information matrix: $G = \begin{bmatrix} 8.73 & 0.85 \\ 0.85 & 0.84 \end{bmatrix}$

Geodesics equation system:

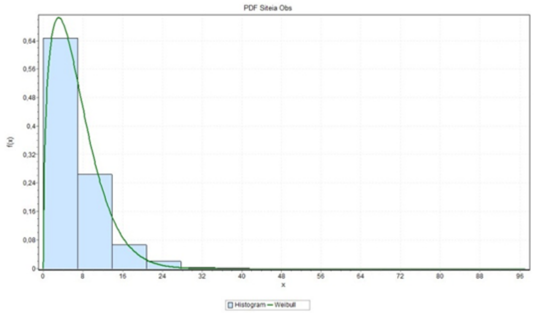
$\omega_1'' - 0.69(\omega_1')^2 + 1.51\omega_1'\omega_2' - 0.30(\omega_2')^2 = 0$

$\omega_2'' - 0.08(\omega_1')^2 + 0.38\omega_1'\omega_2' - 0.75(\omega_2')^2 = 0$

Figure 11c: Distribution fitting for the operational model wind speed results at Mykonos

SITEIA (Wind Speed):

- Observations



Distribution: *Weibull*
Shape parameter: $\alpha = 1.42$
Scale parameter: $b = 7.23$

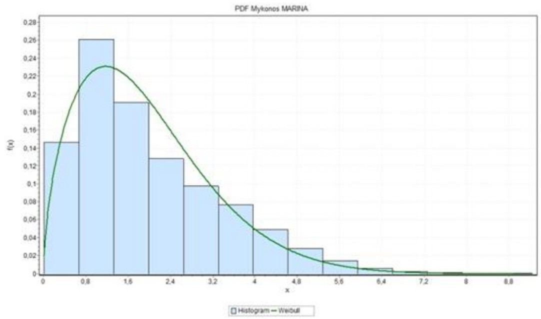
Fisher information matrix: $G = \begin{bmatrix} 105.40 & 3.06 \\ 3.06 & 0.90 \end{bmatrix}$

Geodesics equation system:

$$\omega_1'' - 0.19(\omega_1')^2 + 1.56\omega_1'\omega_2' - 1.18(\omega_2')^2 = 0$$
$$\omega_2'' - 0.01(\omega_1')^2 + 0.10\omega_1'\omega_2' - 0.78(\omega_2')^2 = 0$$

Figure 12a: Distribution fitting for the observations of wind speed at Siteia

- MARINA



Distribution: *Weibull*
Shape parameter: $\alpha = 1.84$
Scale parameter: $b = 2.92$

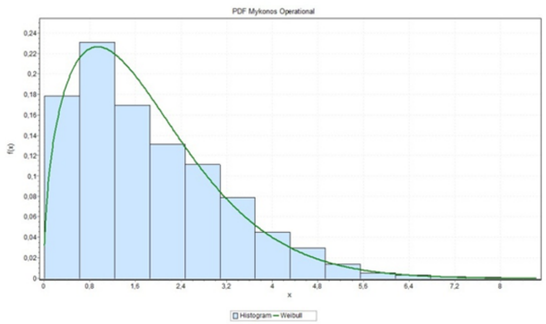
Fisher information matrix: $G = \begin{bmatrix} 28.87 & 1.24 \\ 1.24 & 0.54 \end{bmatrix}$

Geodesics equation system:

$$\omega_1'' - 0.50(\omega_1')^2 + 1.21\omega_1'\omega_2' - 0.22(\omega_2')^2 = 0$$
$$\omega_2'' - 0.07(\omega_1')^2 + 0.32\omega_1'\omega_2' - 0.60(\omega_2')^2 = 0$$

Figure 12b: Distribution fitting for the MARINA project wind speed modeled data at Siteia

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 1.97$
Scale parameter: $b = 3.36$

Fisher information matrix: $G = \begin{bmatrix} 43.81 & 1.42 \\ 1.42 & 0.47 \end{bmatrix}$

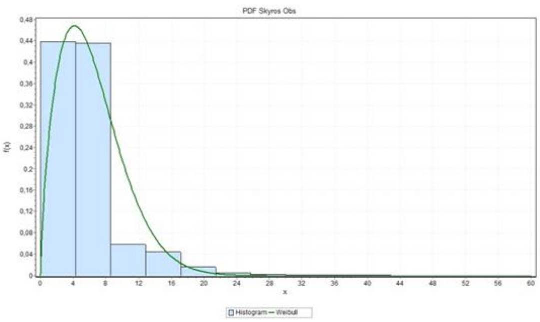
Geodesics equation system:

$$\omega_1'' - 0.45(\omega_1')^2 + 1.13\omega_1'\omega_2' - 0.21(\omega_2')^2 = 0$$
$$\omega_2'' - 0.07(\omega_1')^2 + 0.30\omega_1'\omega_2' - 0.56(\omega_2')^2 = 0$$

Figure 12c: Distribution fitting for the operational model wind speed results at Siteia

SKYROS (Wind Speed):

- Observations



Distribution: *Weibull*
Shape parameter: $\alpha = 1.67$
Scale parameter: $b = 7.10$

Fisher information matrix: $G = \begin{bmatrix} 140.59 & 3.00 \\ 3.00 & 0.65 \end{bmatrix}$

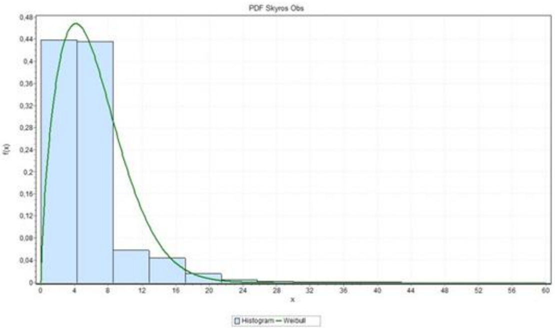
Geodesics equation system:

$\omega_1'' - 0.20(\omega_1')^2 + 1.33\omega_1'\omega_2' - 0.71(\omega_2')^2 = 0$

$\omega_2'' - 0.01(\omega_1')^2 + 0.12\omega_1'\omega_2' - 0.66(\omega_2')^2 = 0$

Figure 13a: Distribution fitting for the observations of wind speed at Skyros

- MARINA



Distribution: *Weibull*
Shape parameter: $\alpha = 1.67$
Scale parameter: $b = 7.10$

Fisher information matrix: $G = \begin{bmatrix} 140.59 & 3.00 \\ 3.00 & 0.65 \end{bmatrix}$

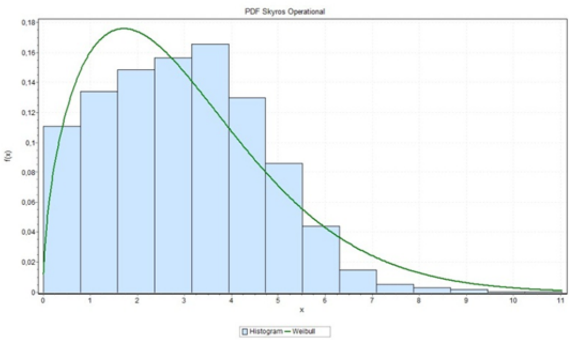
Geodesics equation system:

$\omega_1'' - 0.20(\omega_1')^2 + 1.33\omega_1'\omega_2' - 0.71(\omega_2')^2 = 0$

$\omega_2'' - 0.01(\omega_1')^2 + 0.12\omega_1'\omega_2' - 0.66(\omega_2')^2 = 0$

Figure 13b: Distribution fitting for the MARINA project wind speed modeled data at Skyros

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 1.54$
Scale parameter: $b = 3.36$

Fisher information matrix: $G = \begin{bmatrix} 26.77 & 1.42 \\ 1.42 & 0.77 \end{bmatrix}$

Geodesics equation system:

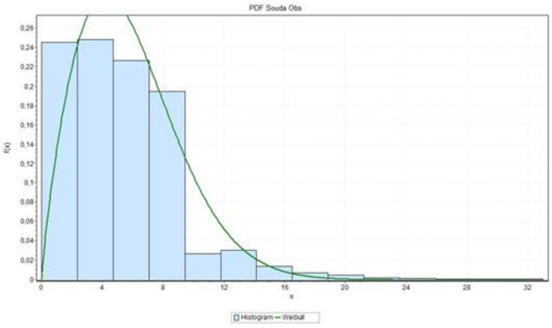
$\omega_1'' - 0.42(\omega_1')^2 + 1.44\omega_1'\omega_2' - 0.43(\omega_2')^2 = 0$

$\omega_2'' - 0.03(\omega_1')^2 + 0.24\omega_1'\omega_2' - 0.72(\omega_2')^2 = 0$

Figure 13c: Distribution fitting for the operational model wind speed results at Skyros

SOUDA (Wind Speed):

- Observations



Distribution: *Weibull*
Shape parameter: $\alpha = 1.83$
Scale parameter: $b = 6.54$

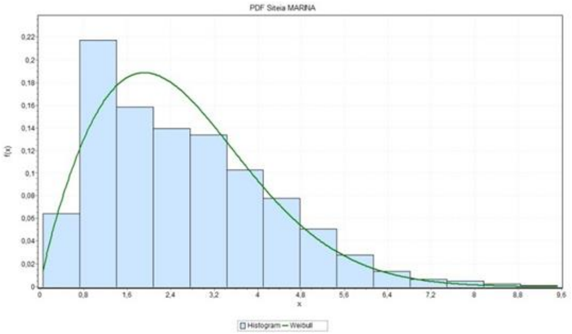
Fisher information matrix: $G = \begin{bmatrix} 143.24 & 2.77 \\ 2.77 & 0.54 \end{bmatrix}$

Geodesics equation system:

$$\omega_1'' - 0.22(\omega_1')^2 + 1.21\omega_1'\omega_2' - 0.50(\omega_2')^2 = 0$$
$$\omega_2'' - 0.01(\omega_1')^2 + 0.14\omega_1'\omega_2' - 0.61(\omega_2')^2 = 0$$

Figure 14a: Distribution fitting for the observations of wind speed at Soda

- MARINA



Distribution: *Weibull*
Shape parameter: $\alpha = 1.93$
Scale parameter: $b = 3.14$

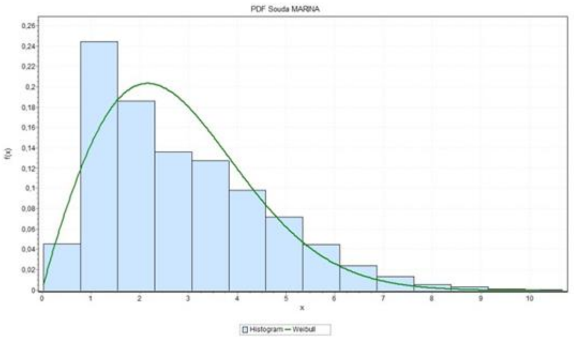
Fisher information matrix: $G = \begin{bmatrix} 36.73 & 1.33 \\ 1.33 & 0.49 \end{bmatrix}$

Geodesics equation system:

$$\omega_1'' - 0.48(\omega_1')^2 + 1.15\omega_1'\omega_2' - 0.20(\omega_2')^2 = 0$$
$$\omega_2'' - 0.07(\omega_1')^2 + 0.32\omega_1'\omega_2' - 0.57(\omega_2')^2 = 0$$

Figure 14b: Distribution fitting for the MARINA project wind speed modeled data at Souda

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 1.93$
Scale parameter: $b = 2.85$

Fisher information matrix: $G = \begin{bmatrix} 30.26 & 1.21 \\ 1.21 & 0.49 \end{bmatrix}$

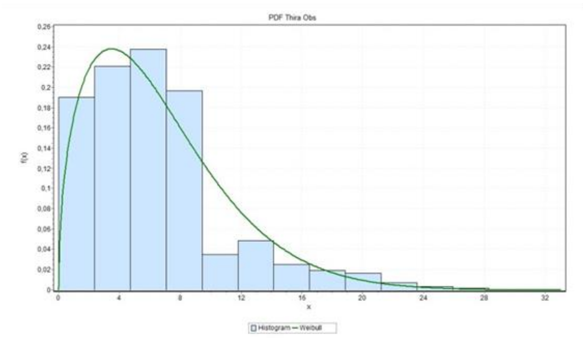
Geodesics equation system:

$$\omega_1'' - 0.53(\omega_1')^2 + 1.15\omega_1'\omega_2' - 0.19(\omega_2')^2 = 0$$
$$\omega_2'' - 0.09(\omega_1')^2 + 0.35\omega_1'\omega_2' - 0.57(\omega_2')^2 = 0$$

Figure 14c: Distribution fitting for the operational model wind speed results at Souda

THIRA (Wind Speed):

- Observations



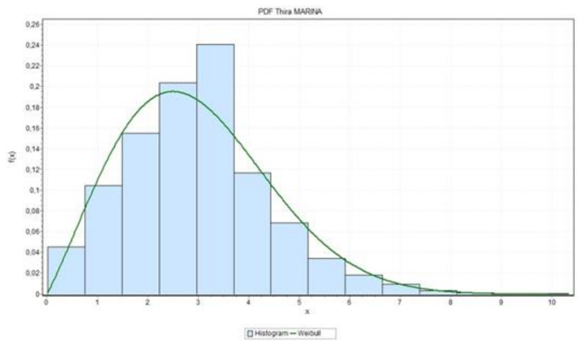
Distribution: *Weibull*
Shape parameter: $\alpha = 1.48$
Scale parameter: $b = 7.37$

Fisher information matrix: $G = \begin{bmatrix} 118.98 & 3.12 \\ 3.12 & 0.83 \end{bmatrix}$

Geodesics equation system:
$$\omega_1'' - 0.19(\omega_1')^2 + 1.50\omega_1'\omega_2' - 1.07(\omega_2')^2 = 0$$
$$\omega_2'' - 0.01(\omega_1')^2 + 0.10\omega_1'\omega_2' - 0.75(\omega_2')^2 = 0$$

Figure 15a: Distribution fitting for the observations of wind speed at Thira

- MARINA



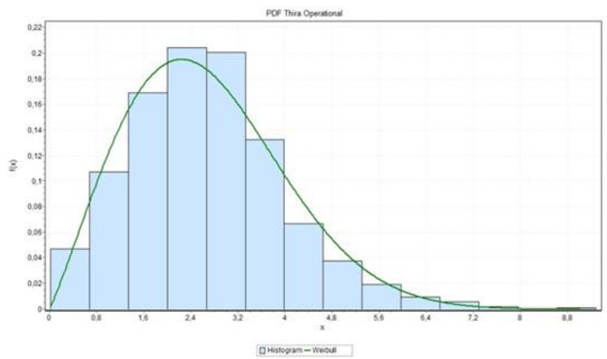
Distribution: *Weibull*
Shape parameter: $\alpha = 2.12$
Scale parameter: $b = 3.37$

Fisher information matrix: $G = \begin{bmatrix} 51.04 & 1.43 \\ 1.43 & 0.491 \end{bmatrix}$

Geodesics equation system:
$$\omega_1'' - 0.46(\omega_1')^2 + 1.05\omega_1'\omega_2' - 0.17(\omega_2')^2 = 0$$
$$\omega_2'' - 0.09(\omega_1')^2 + 0.32\omega_1'\omega_2' - 0.52(\omega_2')^2 = 0$$

Figure 15b: Distribution fitting for the MARINA project wind speed modeled data at Thira

- Operational



Distribution: *Weibull*
Shape parameter: $\alpha = 2.12$
Scale parameter: $b = 3.02$

Fisher information matrix: $G = \begin{bmatrix} 40.99 & 1.28 \\ 1.28 & 0.41 \end{bmatrix}$

Geodesics equation system:
$$\omega_1'' - 0.51(\omega_1')^2 + 1.05\omega_1'\omega_2' - 0.15(\omega_2')^2 = 0$$
$$\omega_2'' - 0.11(\omega_1')^2 + 0.36\omega_1'\omega_2' - 0.52(\omega_2')^2 = 0$$

Figure 15c: Distribution fitting for the operational model wind speed results at Thira

SWH (m)	Operational	MARINA	Observations

	Weibull Shape & Scale Parameters					
	a	b	a	b	a	b
Crete	1,6378	0,8052	1,7351	0,88351	1,6031	1,5076
Lesvos	1,3206	0,67934	1,4684	0,80599	1,5699	0,8774
Mykonos	1,2173	0,9435	1,3028	1,0078	1,4404	1,0306

Table 2: Weibull shape and scale parameters (a, b) for significant wave height (SWH).

Wind Speed(m/s)	Operational		MARINA		Observations	
	Weibull Shape & Scale Parameters					
	a	b	a	b	a	b
Argostoli	1,773	2,826	1,7812	2,881	1,9831	3,335
Chios	1,279	1,9211	1,5585	2,1483	1,8345	3,3878
Kerkyra	1,7636	3,0291	1,7244	3,1471	2,1004	6,1685
Milos	2,0267	3,2124	2,0265	3,2535	1,0962	6,8998
Mykonos	1,476	2,0166	1,5972	2,1748	1,5453	9,3228
Siteia	1,9768	3,3694	1,843	2,9268	1,423	7,249
Skyros	1,5431	3,3638	1,5631	3,6655	1,6768	7,1089
Souda	1,9299	2,8594	1,9386	3,1445	1,8311	6,5426
Thira	2,1216	3,0277	2,126	3,3736	1,4857	7,3702

Table 3: Weibull shape and scale parameters (a, b) for wind speed.

Conclusions

This report summarizes the main activities and results of the WP2 that focuses on the development of novel mathematical tools targeting to the optimum monitoring and minimization of the biases emerging in atmospheric and wave numerical prediction models. The key issue in the presented approach is the utilization of techniques and results obtained in a relatively new branch of Mathematics, the Information Geometry, which allows the use of Riemannian geometry in the optimization procedure avoiding classical simplification adopted by the usually employed least-square methods.

In particular, Information Geometry recognizes families of distributions and data sets as manifolds on which geometrical entities such as Riemannian metrics, distances, curvature and affine connections can be naturally introduced. In this way, distances between different data sets are defined by the Riemannian inner product matrices and the geodesics which are differential equations whose solutions provide the curves of minimum length.

In this report, data obtained within the framework of WP 1, covering the major area of Greece and corresponding to different climatological characteristics, are analyzed. More precisely, wind speed and significant wave height data are studied from three different sources:

-- Observations as recorded by buoy

-- Model simulations in which any available observations are assimilated (hindcasting) from the FP7 program MARINA - http://forecast.uoa.gr/proj_marina.php

-- Operational forecasts from the wind and wave modeling system of the Atmospheric Modeling and Weather Forecasting Group, Physics Department, University of Athens.

The results obtained provide for each site and parameter:

- The probability density functions that optimally fit the data under study, and therefore the statistical manifold in which they are categorized
- The Fisher information matrix that characterizes the geometry of the statistical manifold
- The differential equation (geodesic) that defines the minimum length curves.

These results will be utilized in the next phases of the project for optimization and bias reduction techniques.

Some points that need to be underlined at this stage:

➔ The Weibull distributions fit well both to wind speed and significant wave height data

➔ The obtained shape and scale parameters have a non-trivial spatial variation, a fact that reveals the necessity of locally adapted optimization methods.

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